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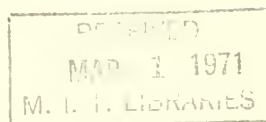
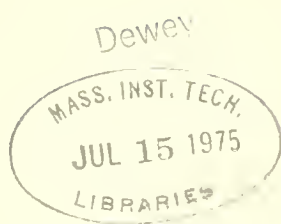
AN INTRODUCTION TO APPLIED MACROECONOMICS

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AN INTRODUCTION TO APPLIED MACROECONOMICS

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* Forthcoming

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PREFACE

We feel that any intermediate or advanced course in macroeconomics should provide some understanding of macroeconometric models. We have both sought to expose our students to models of the U.S. economy. These attempts were invariably disappointing. As an outlet for these frustrations we have felt compelled to write this book, which is designed as a supplement to intermediate-level courses in macroeconomics or applied econometrics.

Most of the macroeconometric models that appear in the literature, especially in the journal literature, are designed to forecast GNP and its components. Their primary purpose is not to illustrate the structure of the economy, but rather to follow its movement over time as closely as possible. Consequently, numerous compromises are made. Logically endogenous quantities are often taken as exogenous, and many equations have only the most tenuous theoretical justification. Explanations of these models tend to be brief and oriented towards the practicing professional forecaster. Sources of data are rarely listed, and weaknesses in the models are rarely acknowledged. Efforts were made to introduce students to macroeconometric models with Daniel Suits' article (1962) and the OBE model (Liebenberg et al, 1966), but for the reasons just mentioned neither paper was adequate to the need.*

At the other extreme are the large "structural" models, of which the Brookings-SSRC effort is best known, (Duesenberry et al, 1965 and 1969). Most equations in these models are the product of careful thought and detailed investigation. Any single sector can be usefully examined at some length.

*References cited here are listed at the end of Chapter I.

But this is just the problem. It is difficult to assign the original Brookings volume in one semester and do anything else.

Two presentations of econometric models have been designed largely for pedagogic purposes. Gregory Chow's simple model (1967) is too simple; it does not provide enough insight into the structure of the economy. Michael Evans' recent book (1969) tries to build an empirically-oriented macroeconomics course around the Wharton EFU forecasting model. We have serious reservations about this model, chiefly because it is a forecasting model. One problem with Evans' book that bears directly on our volume is that anyone wishing to use it can hardly assign another text. Evans provides macroeconomic theory and a macro model, and they are quite closely related. It seems useful to provide a model by itself, designed to complement whatever other materials the instructor wishes to assign.

We thus believe that a large gap exists in macroeconomics texts, and this book seeks to fill that gap. We have estimated a medium-sized quarterly model of the U.S. economy. It is designed mainly to be a teaching tool. We have tried to illustrate the state of the model building art with maximum theoretical and quantitative simplicity consistent with our view of reality. This model is novel in a few respects, most noticeably in its fixed investment^{and wage-price} equations. There are places where the state of the art is not much to brag about, and we have indicated where this is the case. Our model has numerous unsatisfactory equations, as do all macro models, but we emphasize

our weaknesses. Since our avowed aim is to rely on existing knowledge, unsatisfactory equations reflect on the state of the art sometimes, and at others on our own limitations. We are not trying to sell this model to businesses; our aim is rather to stimulate interest in quantitative research in macroeconomics.

As few macroeconomic texts contain much material on distributed lags, we have included a chapter on this topic. Students must have some familiarity with ordinary least squares regression if they are to understand our discussions of the estimated equations in Chapters III-XI. As there are a variety of introductory econometrics texts on the market, we do not cover this topic. Two good introductory treatment are Edwin Kane (1968) and Wonnacott and Wonnacott (1970, Part I).

It is a sad commentary that it is almost impossible to build a model of the U.S. economy without some reliance on unpublished data. To those in and out of the federal government who furnished us with such series, we are extremely grateful. We do not list individuals' names in order to save them from an avalanche of requests, but the text does indicate the agency that supplied each set of figures. Most presentations of econometric models do not indicate data sources very carefully; we have tried to be an exception to this rule.

This model was constructed and simulated on the TROLL time-sharing system at MIT. We are grateful to the TROLL staff, headed by Mark Eisner, for helping us to use this exceedingly powerful tool.

We are indebted to the Edwin Land Foundation for considerable financial support. Daniel Luria was our capable research assistant for two summers. This model truly could not have been built without him. Edward Hyman, Stephen Fisher, and Walter Maling also provided able assistance.

CHAPTER I
INTRODUCTION AND OVERVIEW

Purpose of the Text

This text presents a medium-sized quarterly econometric model of the U.S. economy. The model can be used to make GNP forecasts, but it was designed primarily to illustrate the structure of the economy. Also, we hope to convey to the reader an idea of the state of the art of macro-econometric modeling. The model was constructed as a teaching tool, and it is presented here for the first time.

What is an econometric model? Econometrics is often described as the measurement of relationships suggested by economic theory so that an econometric model amounts to an economic model with numbers that relate it to the real world. ¶ An example will illustrate. The simplest textbook macroeconomic model is the following:

$$(1.1) \quad C = a + b Y$$

$Y = C + I + G$, where a and b are constants, and

C = Consumption Expenditure

I = Investment Expenditure

G = Government Expenditure

Y = Gross National Product.

← It is usually assumed that the values of I and G are determined outside the model. Variables for which this is true are called exogenous variables. Given values for I and G, model (1.1) determines values for C and Y. These latter quantities are the endogenous variables in this model. The first of the equations in (1.1) is a behavioral equation, an attempt to model behavior. The second equation is an identity, or accounting definition.

The form of this model is, of course, based on Keynesian income analysis. This theory suggests the existence of a stable relationship between consumption and income. If this relation is not known or assumed to be stable, model (1.1) has no content. Unless we have reason to believe that a and b are really constant, it is pointless to use this model for predicting the endogenous variables. Thus economic theory must be employed to determine the basic forms of the behavioral equations; it must be the source of hypotheses about what sort of behavioral relationships are stable over time.

As it stands, (1.1) is capable of generating every conceivable prediction, unless something is known or assumed about the values of the constants a and b. According to standard theory, both are positive and b is less than one. The model in this form serves to illustrate the consumption multiplier, but it provides only qualitative information and hence little precise guidance for a decision-maker. To make (1.1) more useful, and to transform it into an econometric model, we might use the history of the economy to estimate the values of the two constants. Once estimates of these quantities are obtained, we could compute a numerical value for the multiplier.

Of course, (1.1) is too simple to use in practice, even with numerical estimates of its coefficients. Any real economy is a good deal more complicated. We surely must attempt to explain changes in investment, and we must allow for lags in behavior. If the aim of a model is forecasting, it can possibly be as small as four equations,^{*} but if it is desired to capture the structure of a developed economy, experience indicates that twenty equations is surely a lower limit. Some models employ more than two hundred equations in an effort to describe the economy more exactly.^{**}

Our model is a compromise. It has 75 equations which determine 75 endogenous variables. We have tried to illustrate the state of the art, with minimum necessary complexity. The model is shot through with imperfections, as are all models.

Chapter II is an introduction to the sorts of assumptions made about the dynamics of behavior in this model and in much of the macroeconomic literature. Some knowledge of its contents will be useful in reading later chapters. The reader should also have some knowledge of ordinary least squares regression, the estimation method used in building this model. We do not discuss the interpretation of least squares estimates in the book; the reader is referred to Kane's text (1968) or to its competitors.

The various sectors of the model are explained in Chapters III-XI. Chapter XII presents the total model and examines some properties of the completed model. Appendix A on Variables and Appendix B on the estimated equations should be useful as references to Chapter XII.

* See Friend and Taubman(1969), for instance.

** See Duesenberry, et al (1965,1969) for a very large model. Nerlove (1966) provides a useful survey of macroeconometric models.

Some Model Building Preliminaries

It was our original intention to use only readily-available data in our model. This proved impossible, as it has for other model-builders. We have indicated the sources of published data and the agencies supplying unpublished figures. We have tried to provide more complete documentation of this sort than is standard in the presentation of macroeconometric models.

We have attempted to make our model a self-contained system, to use only logically exogeneous quantities as exogenous. This has led us to make compromises in our specification of the variables present in behavioral equations in many cases. For instance, the yield on common stocks can reasonably be expected to influence business investment decisions. But it is not currently possible to explain statistically the movements in common stock prices. Were we to rely on this variable, we would be compelled either to tolerate a very weak equation that explained it, or to treat it as exogenous. As neither course seemed very satisfactory, this variable does not appear in the model. Other variables are employed to capture its effects. Some quantities, like Inventory Valuation Adjustment, are logically endogenous and essential to the model. These we explain, but with very weak equations. This desideratum, the need to explain any logically endogenous variable appearing in the model, is not present in studies of individual sectors or of single variables. But it is critical to properly modeling an entire economy. We have tried to make our specifications as theoretically sound as we possible could, even in the face of this problem, but the job was not easy.

These precepts clearly imply what the reader will soon discover for himself, that econometric model building is an art, and not a refined one at that. Economic data is often poor and ill-suited to testing hypotheses or estimating parameters. Economic theory varies enormously in its ability to provide clearcut guidance. Consumption theory provides clearcut testable hypotheses upon which we have put our reliance. Investment theory and wage theory, however, are in a confused state so that we were more at liberty to experiment with moderately novel, although theoretically standard formulations of our own. Since our intent throughout has been to build a model that will have maximum theoretical acceptability subject only to a need for relative simplicity, departure from received theory has been limited to sectors where the doctrine itself is not widely accepted. To the extent that we have successfully adhered to these principles, this econometric model can be viewed as a status report about the condition of macroeconometric theory and its econometric implementation.

As mentioned above, all behavioral equations in this model were estimated by ordinary least squares. Data from the period 1954I - 1967IV were employed. When the equations to be estimated are part of a system of simultaneous equations, as in this model, least squares estimates are known to be biased and inconsistent. We have used this method, rather than alternative approaches which eliminate bias in large samples, for two reasons. First, least squares is simpler and easier to interpret than the alternatives. This seems to us to be quite important in a textbook.* Second, seldom do

* Furthermore, in small samples, many arbitrary decisions must go into selection of instrumental variables so that students interested in reproducing and then modifying our calculations can only do so initially with ordinary least squares estimates.

macroeconometrics least squares estimates differ much from estimates produced by consistent estimation methods. Estimates of the model using methods designed for simultaneous systems appear in Appendix C (to be completed March, 1971).

Another statistical problem should be mentioned. When lagged values of the dependent variable values determined in earlier periods are used as independent variables in a regression equation, and when the random error or disturbance terms in different periods are correlated, coefficient estimates are biased. Methods have been devised to correct for this, but we have only rarely employed them. When estimated disturbance terms in adjacent periods are correlated, the specification of the equation is often wrong. Something systematic is happening that is not being captured. Rather than make statistical assumptions about the behavior of what has been omitted, it seemed more sensible to us to acknowledge that the equation is weak. Stated in more technical fashion, we normally prefer to interpret serial correlation of residuals as an indication of mis-specification, not as a sign that nature has generated serially dependent disturbance terms.

This model is not a set of linear equations. Some equations are linear in the variables involved, but many are not. In fact, it would require fundamental changes in the structure of this model to make it linear. To cite only one example, the demand for Gross National Product is computed in current dollars. This is obtained by multiplying the constant dollar figure by the implicit price deflator which, of course, produces a non-linear equation.

Several years ago a medium scale non-linear model would have been a severe computational burden. It would have been difficult to solve the model for the values of the endogenous variables. Now, thanks to modern computer hardware and software, this is no longer true. The TROLL system at MIT, on which this model was constructed, is able to solve large systems of non-linear equations with breathtaking ease.* Non-linear models do raise problems of interpretation, of course. In particular, multipliers are not constant; they depend on the values of all variables. While this is unfortunate, neither we nor the practicing econometrics profession really believe that the economy can be adequately represented by a linear model.

Sector Presentations

Each of Chapters III-XI deals with a sector of the economy and with its representation in this model. Often the line between sectors was rather arbitrary, and we drew it where it seemed most sensible to do so. Chapter XII emphasizes the relations that tie the sectors together.

In our discussion of each sector, we first present and define the variables employed. We then list the relevant identities. Next we discuss the estimates of the behavioral equations, mentioning approaches that failed as well as presenting our best estimates. The rationale for each equation in the model is spelled out as explicitly as seems necessary. We try to indicate the strengths and weaknesses of the individual estimates.

* It takes less than 10 seconds to solve the model for one period on TROLL (0) as implemented on the IBM 7094 computer. See Eisner (1969) for a description of this system.

Each sector is further tested and examined by means of two dynamic simulations. We shall now indicate what these are and how their output is to be interpreted.* Consider the following dynamic version of model (1.1):

$$(1.2) \quad C = a + bY + cC(t-1)$$

$$Y = C + I + G$$

This is identical to (1.1) except for the presence of the lagged endogenous variable $C(t-1)$, the previous period's consumption spending. Such lagged endogenous quantities are called predetermined variables. Hereafter, lagged values are denoted by time subscripts while the current period subscript will be omitted. Given the values of $C(t-1)$, I , and G , the model can be solved for current period values of C and Y .

A dynamic simulation of model (1.2) would proceed as follows. Take the actual value of C in period zero and the values of I and G in period one, and solve the model for estimates of C and Y in period one. Then take the actual values of I and G in period two and the estimated value of C in period one and solve for estimates of C and Y in period two. The simulation proceeds iteratively in this fashion for as many periods as desired. The essence of a dynamic simulation is that the model generates its own lagged endogenous variables.

The individual sectors and the entire model are simulated over the period 1954I-1967IV, the period to which the behavioral equations were fitted. The estimated values of the endogenous variables are compared to the actual values. This simulation accomplishes two things. First, it gives a better idea than just regression statistics of the quality of the various equations. Second, it indicates whether any equations have a tendency to go off the track, to generate estimates that are consistently too high or too low.

* See also Fromm and Taubman (1968).

Tables in each chapter present the average value of each of the endogenous variables over this period. This information is useful in its own right to give one a sense of the importance of the various variables. Second, we present the root-mean-squared (RMS) error for each variable, a basic tool for analyzing the dispersion of any variable. The error in each period is defined as the value computed by the simulation minus the actual value of the variable. A negative error is thus an under-estimate. The RMS error is defined as the square root of the average of the squared errors. The RMS error is also exhibited as a percentage of the mean value of the variable to allow for differences in scale among variables.* The absolute and relative size of the RMS error gives an intuitively useful idea of the quality of the estimate.

To see if variables are systematically over- or under-predicted, the arithmetic average of the errors is presented. In interpreting this figure, remember that a positive quantity means that the model over-estimated the variable in question on average. We also present the results of a simple t-test of the hypothesis that this mean error equals zero. This is by no means an exact test for data generated in this manner, but rejection of the hypothesis that the mean error is zero provides some indication that the variable has strayed systematically.

Besides the 1954 - 1967 simulation, we have simulated each sector and the model as a whole over the four quarters of 1968. The statistics just discussed are presented for these simulations also. The purpose of these

* When the variable can assume negative as well as positive values, this ratio is no longer very useful.

CHAPTER I: Introduction (includes Preface ref's.)

Basic Concepts of Models

Christ, C. F. Econometric Models and Methods, New York: John Wiley and Sons, 1966. Chapter I.

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Suits, D. B. "Forecasting and Analysis with an Econometric Model," American Economic Review, 52(March, 1962): 104-132.

Econometric Methods

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Wonnacott, R. J., and Wonnacott, T. H. Econometrics, New York: John Wiley & Sons, 1970.

runs is to see how well the model performs outside the sample period. Large RMS errors or large t-statistics generally indicate specification problems.

Model builders, including ourselves, are inclined to "mine the data", i.e., to estimate a variety of different equation forms until one is found with acceptable error variance, (i.e., small) coefficient signs and t-statistics. Tests made outside the period of fit provide some insurance against this abuse of statistical theory (however compelling the procedure appears to be), although four data points by no means provide air-tight assurance.

Chapter I, (cont'd).

Model Simulation and the TROLL System

Eisner, M. Manual for the Troll System, Massachusetts Institute of Technology, 1969 (mimeographed).

Fromm, G., and Taubman, P. Policy Simulations with an Econometric Model, Washington: The Brookings Institution, 1968.

CHAPTER II

Distributed Lags

Introduction

Much economic theory is static; time does not enter in an essential way. Static theory seldom provides all the information needed to model the real world. Suppose that the static theory says that some variable Y depends on another variable, X. If the quantity Y represents the outcome of a decision process, such as total consumption and total investment, it is unlikely that changes in X will be immediately reflected in Y. It is often quite important, especially for policy decisions, to be able to characterize the lag involved, to determine how long it takes Y to respond to changes in X.*

Individual decision-makers normally respond to changes in X some time after they occur, but not all people will wait the same length of time to act. If they did, changes in Y would lag changes in X by some fixed length of time. If individuals' lags differ, the aggregate response to changes in X will be spread over more than one period of time. Such lags are called distributed lags**. These may exist at the individual level as well as in the aggregate if individuals consider more than one lagged value of X in making decisions. Distributed lags at the household level are suggested by the Permanent Income hypothesis of consumer spending behavior, for instance.

* Much current controversy about the exercise of monetary and fiscal policy centers on the length and stability of lags in the effect of changes in the quantity of money or taxes on the aggregate variables they presumably influence.

** A useful survey of much of the material covered in this chapter is Z. Griliches, "Distributed Lags: A Survey", Econometrica, XXXV (January, 1967), 16-49.

If Y is determined by X through a distributed lag, this means that Y depends on more than one past value of X. If we assume a linear model, we can write the general distributed lag relation as

$$(2.1) \quad Y(t) = a[w_0X(t) + w_1X(t-1) + w_2X(t-2) + \dots]$$

The w's add to one.* It is sometimes assumed that they are all positive, although sensible distributed lags may arise where some weights are negative. There may or may not be an infinite number of w's. (The difference between a rapidly diminishing infinite series and a finite series is difficult to detect in practice, and the former is often simpler to manipulate algebraically.) The constant a represents the eventual change in Y that occurs in response to a maintained unit change in X.

In what follows, we will be interested in two aspects of distributed lags. The first¹ concerns the relation among alternative lag specifications and some of their statistical aspects. The other concern is with ways of obtaining analytical insight into the dynamic properties of an already estimated relationship.

In the real world we must deal with, there is never enough good-quality economic data to permit direct statistical estimation of a large and perhaps infinite number of w's. Some assumptions about the shape of the sequence of w's must then be made; the sequence must be described by a few parameters so the parameters can be estimated from data. If the initial

* a can be chosen so that the sum of the w's add to one simply by dividing each original lag coefficient by the sum of these coefficients and multiplying the entire expression in square brackets involving these normalized coefficients by that sum. The significance of this normalization is discussed in the final section of this chapter.

impact is greatest and falls off thereafter, the sequence of weights can be economically described by the two parameters defining a geometric series. Such a sequence is called the Koyck lag distribution. Another common response is to have a small initial impact which gradually builds up to a peak and then falls off to small values once more. Simple two or three parameter distributions exist that can readily describe this pattern of lag weights. Theoretical analysis can then proceed in terms of the parameters of the sequence of w 's.

One approach is to assume that the sequence of w 's can be adequately approximated by a polynomial function of the lag involved. If you then specify the degree of the polynomial, the coefficients of the polynomial can be estimated. For instance, one might assume that

$$(2.2) \quad w_i = a + b i + c i^2, \text{ for } i = 0, 1, \dots, 8.$$

and that $w_i = 0$ for i greater than or equal to nine. Using a computational method proposed by S. Almon, (1965), the coefficients a , b and c can be estimated statistically. The main advantage of this method is that reasonable prior restrictions imposed on the lag distribution greatly reduce the number of coefficients that must be estimated.

This approach has the drawback that you must specify a priori the number of non-zero w 's and the degree of the polynomial. One ends up with an extensive search for the "best" combination, without any clear cut theoretical statistical guidance. Also, if seven lagged X 's are assumed to influence Y , one must begin estimation with the eight observation on Y . With long lags and few available observations, a good deal of information may be lost this way, although the same problem arises to some extent in all lag estimation methods. These polynomial lags may be useful when the

lag is known to be finite, and where many observations are available. We employ this technique in the Appendix to Chapter IV and in Chapter V."

The second and more common approach throughout this model as well as in other econometric work is to assume that there are an infinite number of non-zero w_i 's. For this to make any sense, w_i has to fall rapidly to zero as i , the lag index, becomes large. The simplest assumption of this sort is

$$(2.3) \quad w_i = (1-k)k^i, \text{ for } i = 0, 1, 2, \dots$$

Here k must be a constant between zero and plus one in order for the sum of the w_i to equal one, since k^i is a geometric progression which sums to unity. Notice that all the w_i are positive. This assumption is reasonable in many situations, but not all: the matter is discussed fully elsewhere, although interpretation of lag properties is greatly facilitated by both assumptions. The assumptions about w_i expressed by equation (2.3) were first proposed and explored by Koyck (1954) and we speak of this as a first-order or geometric or Koyck distributed lag.

The beauty of this lag structure and its more complicated variants is that equation (2.1) can be rewritten so as to involve $X(t)$ and a few lagged values of Y . In fact, if there are N parameters like k in (2.3) that determine the lag structure, (2.1) can be rewritten to involve just N lagged values of Y .

In the next section, we shall examine the properties of the Koyck lag structure in detail. While this lag structure is basically

simple, it is a natural vehicle to use to explain techniques which are useful in analyzing more complex structures.

a. The Koyck Lag

We shall first verify that assumption (2.3) permits us to drastically simplify equation (2.1). Substituting (2.3) into (2.1) we obtain

$$\begin{aligned}
 (2.4) \quad Y(t) &= a(1-k) \sum_{i=0}^{\infty} k^i X(t-i) \\
 &= a(1-k)X(t) + a(1-k) \sum_{i=1}^{\infty} k^i X(t-i).
 \end{aligned}$$

Notice that the smaller is k , the more rapidly the influence of past X 's decays. Lagging (2.4) by one period and multiplying by k , we have

$$\begin{aligned}
 (2.5) \quad kY(t-1) &= a(1-k)k \sum_{i=0}^{\infty} k^i X(t-1-i) \\
 &= a(1-k) \sum_{i=1}^{\infty} k^i X(t-i)
 \end{aligned}$$

Subtracting (2.5) from (2.4), we have

$$(2.6) \quad Y(t) = a(1-k)X(t) + k Y(t-1).$$

Two properties of this equation are immediately obvious.

If $X(t)$ is increased by one unit, $Y(t)$ will rise by a $(1-k) = a w_0$ units. If X is increased by one unit and is maintained at the new level forever, $Y(t)$ must eventually equal $Y(t-1)$, provided k is less than one in absolute value. Setting $Y(t)$ equal to $Y(t-1)$ and solving, we obtain $Y(t) = aX(t)$. Both these properties agree with equation (2.1). Further, we shall verify below that if k is between zero and one, a maintained change in X will cause Y to steadily approach its new equilibrium. The larger is k , the more important is past history, and the slower Y approaches equilibrium.

We shall now examine parameters used to summarize lag distributions, and we shall evaluate these quantities for the Koyck lag structure. Clearly this structure is easily summarized by the parameter k , but the summary parameters we shall consider and (especially) the way we shall find them will be useful in the consideration of more complex lag structures.

Adjustment Time and Median Lag

In equation (2.6), suppose that $Y(0) = aX(0)$. That is, assume that the system is in equilibrium in period zero. Suppose $X(1) = X(0) + 1$, and that this value of X is maintained thereafter. Then $Y(1) = a(1-k)$

$$[X(0) + 1] + kY(0) = Y(0) + a(1-k).$$

Substituting further, we find

$$\begin{aligned} Y(2) &= (1-k)Y(0) + a(1-k) + k[Y(0) + a(1-k)] \\ &= Y(0) + a(1-k)(1+k), \\ Y(3) &= (1-k)Y(0) + a(1-k) + k[Y(0) + a(1-k)(1+k)] \\ &= Y(0) + a(1-k)(1+k+k^2), \end{aligned}$$

and in general, summing the geometric series in k ,

$$(2.7) \quad Y(t) = Y(0) + a(1-k^t).$$

The new equilibrium value of Y will be $Y_e = Y(0) + a$, as asserted above.

Notice that if k is between zero and one the difference between $Y(t)$ and Y_e declines steadily over time, also as asserted above.

The fraction of the adjustment to the new equilibrium completed after t periods is simply

$$\frac{Y(t) - Y(0)}{Y_e - Y(0)} = \frac{a(1-k^t)}{a} = (1-k^t).$$

The median lag, T_{md} , is simply that value for t for which the fraction of adjustment completed equals one half. Thus we have

$$(2.8) \quad .5 = 1 - k^{T_{md}}, \text{ or } T_{md} = \log (.5) / \log (k).$$

Note that as k goes to zero, so does the median lag, as one might expect.

The median lag is less than or greater than one period, depending if k is less than or greater than .5:

In complicated lag structures, the median lag may be hard to compute. In its place, we usually use the mean lag, T_m , to measure the speed of response. The mean lag (for all $w_i \geq 0$) is defined by

$$(2.9) \quad T_m = \sum_{i=0}^{\infty} i w_i.$$

Lag Operators and Lag Polynomials

Before computing the mean lag for the Koyck case, it will be useful to introduce two concepts that are of broad application, lag operators and lag polynomials. The first is the lag operator, which we shall write as L . This operator is defined by the following identity. Let Z be any time-series variable:

$$(2.10) \quad L^k Z(t) = Z(t-k),$$

where k is a non-negative integer.

We now proceed to rewrite the general distributed lag equation (2.1) in this notation -

$$(2.11) \quad Y(t) = \left[\sum_{i=0}^{\infty} w_i L^i \right] aX(t) = P(L) aX(t).$$

The second important concept, the lag polynomial, is represented here by the polynomial in brackets, $P(L)$, in Equation (2.11). This equation has a variety of advantages over the equivalent representation (2.1).

Mean Lag

It is a simple matter to obtain the mean lag from this form by differentiating $P(L)$ with respect to L , term by term and then setting

L equal to one. It should be clear that $T_m = P'(1)^*$.

From (2.3), equation (2.11) in the Koyck case becomes

$$(2.12) \quad Y(t) = \left[(1-k) \sum_{i=0}^{\infty} k^i L^i \right] a X(t) \\ = \left[\frac{1-k}{1-kL} \right] a X(t).$$

The second line was obtained from the first by treating L like an ordinary constant between zero and one, and expressing the sum of the geometric series $(kL)^i$ in closed form. Notice that if we multiply both sides of (2.12) by $(1-kL)$ and substitute $Y(t-1)$ for $LY(t)$, we obtain equation (2.6). Differentiating the lag polynomial in the second line of (2.12) with respect to L and setting L equal to one, we obtain:

$$(2.13) \quad T_m = P'(1) = \frac{k}{1-k}$$

As with the median lag, when k goes to zero, the mean lag does also.

Variance of Lag Distribution

The variance of the lag distribution, V_L is another useful magnitude that is obtainable for lag distributions with positive w_i . It expresses how much the influence of X is spread out over time, which the mean or median lag does not do. This quantity is defined by

$$(2.14) \quad V_L = \sum_{i=0}^{\infty} w_i [i - T_m]^2 = \sum_{i=0}^{\infty} i^2 w_i - T_m^2.$$

* The reader should set up a polynomial, follow the rules given and the result follows immediately. This form of generating function analyses has been taken directly from probability analysis. A lucid more complete exposition will be found in Feller, An Introduction to Probability Theory and its Applications, Vol.1 (New York: John Wiley, 1950)

An examination of (2.11) should make it clear that the first term is found by differentiating the lag polynomial twice with respect to L , setting L equal to one, and adding the mean lag. In the Koyck case,

$$\begin{aligned}
 (2.15) \quad V_L &= P''(1) + P'(1) - [P'(1)]^2 \\
 &= \frac{2k^2}{(1-k)^2} + \frac{k}{(1-k)} - \frac{k^2}{(1-k)^2} = \frac{k}{(1-k)^2}
 \end{aligned}$$

Individual Lag Weights

The lag polynomial has one other important use. From it, we can readily derive the lag weights. These would otherwise be difficult to compute in complicated lag structures. Notice that the N^{th} derivative of the lag polynomial in (2.11) with respect to L is given by

$$(2.16) \quad \frac{d^N P(L)}{dL^N} = \sum_{i=N}^{\infty} \frac{i!}{(i-N)!} w_i L^{i-N},$$

since the terms corresponding to i less than N vanish identically. (Recall that $N! = N(N-1)(N-2)\dots 2.1$) Setting $L=0$, all terms with i greater than N vanish.¹ Since $0!$ is identically equal to one, we have the result

$$(2.17) \quad \left. \frac{d^N P(L)}{dL^N} \right|_{L=0} = N! w_N.$$

¹ If the reader will work out an example with a 3rd degree polynomial for $N = 3$, the result shown in (2.17) becomes immediately evident.

We can thus go back uniquely from the lag polynomial to the w 's. This relation is easy to verify in the Koyck case using (2.12) and it is occasionally useful in higher-order structures.

Before examining such structures, it will be useful to illustrate the application of the tools we have developed. Suppose we have estimated a distributed lag relation between Y and X and have obtained the following equation:

$$(2.18) \quad Y(t) = .30 X(t) + .80 Y(t-1).$$

The short-run impact of X upon Y is simply .30. To obtain more information, compare equation (2.18) to equation (2.6). It is clear that $k = .80$ and that a , the long-run impact of X on Y , is equal to $.30/(1-.80) = 1.50$. Using equation (2.8), we can compute the median lag:

$$T_{md}(\log(.5)/\log(.8)) = (-.693)/(-.223) = 3.11 \text{ periods.}$$

From equation (2.13), the mean lag is simply $.80/(1-.80) = 4.00$ periods. Similarly, equation (2.15) could be used to compute the variance of the lag distribution, and equation (2.17) could be employed to compute the individual lag weights.

Now suppose that the estimated relation between X and Y had been

$$(2.19) \quad Y(t) = .20 X(t) + 1.10 Y(t-1).$$

Can we compute similar statistics for this equation? No, since the implied value of k , 1.10, is not consistent with the Koyck lag scheme. It may be possible to make sense of equation (2.19), but it cannot be interpreted as an estimate of a geometrically distributed lag function.

b. More General Lag Mechanisms

We shall begin this brief discussion of more complicated lag schemes with a second-order example. Suppose that the quantity X in (2.12) represents an observed data series, but that Y is not observable. For instance, in an investment study, X might be sales and Y might be decisions to purchase new capital goods. No data on decision to purchase new capital is readily available, but it is desired to explain investment spending, Z . We assume that Z is observable and that it is related to Y according to

$$(2.20) \quad Z(t) = \left[\frac{1-m}{1-mL} \right] b Y(t),$$

where L is the lag operator, as before and m is a constant between zero and one. The mean lag of (2.20) is clearly $m/(1-m)$. In the investment study, this would represent the mean lag between decisions and deliveries. We can combine (2.12) and (2.20) in a way that expresses $Z(t)$ as a function only of the observable variable $X(t)$:

$$(2.21) \quad Z(t) = \left[\frac{(1-m)(1-k)}{(1-mL)(1-kL)} \right] a b X(t).$$

By differentiating the lag polynomial in brackets in (2.21) with respect to L and setting L equal to one, it can be shown that the mean lag in (2.21) is equal to $\left[m/(1-m) \right] + \left[k/(1-k) \right]$. The mean lags add when linear equations are combined in this fashion. The variance of the lag can also be computed, and equation (2.17) could be used to compute the lag weights, the w_i .

Multiplying (2.21) though by $(1-mL)(1-kL)$ and rewriting we obtain*

$$(2.22) \quad Z(t) = \left[(1-m)(1-k)ab \right] X(t) + (k+m) Z(t-1) - km Z(t-2).$$

Suppose we estimate the coefficients of (2.22) from time-series data on $Z(t)$ and $X(t)$. The question may arise whether the estimated coefficients can be interpreted as having come from a dynamic structure that has a monotonic approach to equilibrium. Let our estimated equation be

$$(2.23) \quad Z(t) = A X(t) + B Z(t-1) + C Z(t-2).$$

It can be shown that this function will imply a set of w_i greater than zero, and hence a monotonic approach to equilibrium, if the following conditions are satisfied:

$$(2.24) \quad \begin{aligned} 2 > B > 0, & \quad -1 < C < 1, \\ B + C < 1, & \quad B^2 \geq -4C. \end{aligned}$$

In the general case, distributed lag equations may involve many lagged Z 's, and there may be lagged X 's as well.** The restrictions analogous

* It can be shown that for k and m less than one, all the w_i will be positive. Thus changes in X will cause Z steadily to approach its equilibrium.

** Conditions (2.24) originate from restrictions on the roots of the second order difference equation in $Z(t)$ which must be real and less than unity. The reader is referred to William Baumol, Economic Dynamics, 3rd ed., Chapter for a general treatment of difference equations.

to (2.24) that estimated coefficients must satisfy in order to represent sensible lag structures will then be quite complex; they will not concern us here.

To examine the general case, we define:

$$F(L) = a_0 + a_1 L + a_2 L^2 + \dots + a_m L^m$$

$$G(L) = 1 - b_1 L - b_2 L^2 - \dots - b_n L^n$$

The general difference equation may then be rewritten as a rational distributed lag:

$$(2.25) \quad Z(t) = \left[\frac{F(L)}{G(L)} \right] X(t)$$

This is called a rational lag since the lag polynomial $P(L)$ may be written as the ratio of two polynomials in L .^{*} Notice that the long-run impact of $X(t)$ on $Z(t)$ is given by $F(1)/G(1)$, i.e., it is the change in equilibrium Z brought about by a unit change in X . We can rewrite (2.25) in the same form as (2.21), by dividing $F(L)$ by the scalar $F(1)$ and $G(L)$ by the scalar $G(1)$, so that the coefficients of both lag polynomials normalized in this fashion add to unity; to retain equation (2.25) as written originally, the expression in brackets must be multiplied by $\frac{F(1)}{G(1)}$.

$$(2.26) \quad Z(t) = \left[\frac{F(L)/F(1)}{G(L)/G(1)} \right] \frac{F(1)}{G(1)} X(t) = \left[\frac{G(1) F(L)}{F(1) G(L)} \right] \frac{F(1)}{G(1)} X(t)$$

^{*}Equation (2.21) makes no sense as a distributed lag unless $G(1)$ is positive, because otherwise a stable equilibrium does not exist. Dale Jorgenson, "Rational Distributed Lag Functions", *Econometrica*, XXXIV (January, 1966) 135-149, presents results about rational lag distributions of interest for econometric application and interpretation.

The quantity in brackets is a normalized lag polynomial; the original coefficients have been multiplied by the scalar quantity $G(1)/F(1)$ so that the sum of the w 's is unity. It can be differentiated with respect to L to find the mean lag, the variance of the lag distribution, and the individual lag weights. The normalization is important because it eliminates steady state effects of changes in $X(t)$ on $Z(t)$ and also removes effects of the explanatory variable's measurement units on calculated lag characteristics. An alternative interpretation is that $X(t)$ is rescaled to its steady state value by multiplying by the proper normalization factor $F(1)/G(1)$. A unit change in this normalized variable, say $X^*(t)$, then shows the purely transient response of $Z(t)$ when it undergoes a unit change.

We conclude this chapter with an illustration of the use of the tools developed here. Consider the following estimated equation.

$$(2.27) \quad Y(t) = 1.0 X(t) + 2.0 X(t-1) + 1.10 Y(t-1) - .20 Y(t-2).$$

Conditions (2.24) are satisfied, so the w_i are all positive. The initial impact of X on Y is simply 1.0, while the long-run effect of a change in X is given by $(1.0 + 2.0)/(1.0 - 1.10 + .20) = 30$.

To obtain further results, we rewrite (2.27) in the form of (2.26). Here we have

$$F(L) = 1.0 + 2.0 L; F(1) = 3.0$$

$$G(L) = 1.0 - 1.10 L + .20 L^2; G(1) = .10$$

Hence (2.27) may be written as

$$(2.28) \quad Y(t) = 30 \cdot X(t) \left[\frac{(.10)(1 + 2L)}{(3)(1 - 1.1L + .2L^2)} \right].$$

Differentiating the lag polynomial in brackets with respect to L and evaluating the derivative at $L = 1$, we obtain the mean lag:

$$(2.29) \quad T_m = \frac{.10}{3.0} \cdot \frac{.2 + 2.1}{.01} = 7.7 \text{ periods.}$$

We could compute the variance of the lag distribution similarly, and equation (2.17) could be used to obtain the lag weights. There is an easier way to obtain the initial lag weights in this case, however, and we now illustrate it.

The lag polynomial can be written in the following form after some trivial rearrangement:

$$(2.30) \quad \left[\frac{(1 + 2L)}{30} \right] \left[\frac{1}{[1 - (1.1L - .2L^2)]} \right]$$

The second term is simply the sum of a geometric series.* Writing the series out, we obtain

* Recall the series expansion of $\frac{1}{1-Z} = 1 + Z + Z^2 + Z^3 + \dots$ for $|Z| < 1$.

In this instance, $Z = 1.1L - .2L^2$.

CHAPTER II - Distributed Lags

A Useful General Survey

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Chapter III

Personal Consumption Expenditure

In this chapter, we shall present the consumption sector. Its role is to determine total consumption expenditures in constant dollars, given disposable personal income and the consumption price deflator. Estimates of consumption of services are also required for use in our inventory equation (Chapter V). For this reason, and because the different categories of total consumption exhibit different dynamic properties, four behavioral equations were used to explain consumption by category. We shall first describe the data and then present two commonly-discussed dynamic models of consumer spending behavior. The equations of this sector are then examined. Dynamic simulation results of the consumption sector conclude the chapter.

The Data

The data series used in this sector are listed in Table III.1. The population series, LTPOP, is the quarterly average of monthly figures taken from the Survey of Current Business. The three aggregate series CPTOTD, YDPI, and PCPTOT were taken directly from the Survey. The first two per capita quantities, ND and DUR, were computed from total constant dollar consumption in these categories and LTPOP.

Table III.1

Variables Appearing in the Consumption Sector

(Endogenous - Determined in Consumption Sector)

CND	Per-capita consumption expenditures on non-durables (constant dollars).
CDUR	Per-capita consumption expenditures on consumer durables (constant dollars).
CHS	Per-capita consumption expenditures on housing services (constant dollars).
CNS	Per-capita consumption expenditures on services other than housing services (constant dollars).
CPTOT	Per-capita total consumption expenditures (constant dollars).
CTOT	Total consumption expenditures (billions of constant dollars) (Also defined by identity 3.18)

(Endogenous - Determined Elsewhere in the Model)

YDPI	Disposable Personal Income (billions of current dollars).
PCPTOT	Implicit price deflator for total consumption expenditures (1957-59) = 100
Y	Per-capita Disposable Income (constant dollars). (See identity 3.16).

(Exogenous Variable)

LTPOP	Total population (billions of persons).
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NOTE: All flow variables from the National Income Accounts are seasonally adjusted quarterly totals measured at annual rates.

Implicit deflators for the various categories of services consumption are available in the Survey only on an annual basis, while current dollar spending by category and the deflator for total services are published quarterly. The annual deflator for housing services is nearly a pure trend, however, so we confidently interpolated to obtain a quarterly series. * This permitted us to calculate housing services consumption in constant dollars and, by subtraction, non-housing services in constant dollars. Then LTPOP was used to obtain HS and NS.

Specifications Tested

A number of basic issues had to be resolved before hypothetical consumption equations could be written. The first question is whether or not to include relative prices. It is unlikely that there are substantial price effects as between broad categories of consumer expenditure. Some macro-models find statistically significant price terms, but they contribute little to explained variance. In addition, the consumption deflators are known to be subject to substantial measurement error, and they are not easily forecast in a macro-model. We thus decided to exclude price terms from the consumption equations. Most theory treats consumer demand in real terms, and we have accepted that formulation. Thus consumers are assumed not to suffer from "money illusion", though this assumption has recently been challenged by William H. Branson and Alvin K. Klevorick (1969).

* Interpolation was quadratic except for the last two years of data. The extreme smoothness of the data permitted linear interpolation.

In constant dollars, there has been a shift of expenditure to services and away from non-durables, with the overall average propensity to consume holding steady at about .92. This suggests that the long-run average propensities to spend on the various categories have been shifting. Efforts to capture these shifts by introducing quadratic income terms were unsuccessful.

It would appear that the most sensible measures of consumption are in per capita terms. It should make some difference in aggregate consumption behavior whether a given amount of disposable income is divided among 100 million people or 200 million people, when positive saving exists and is responsive to household size. By working with per capita quantities, we add an element of reality and, as a bonus, remove some of the common trend from the variables.*

Once the decision to work with per capita variables in constant dollars had been made, most of the specification was complete. It still remained to decide on the particular functional forms, and there was some uncertainty as to the nature of the distributed lags involved. To examine the two models that underlay our exploration, we define the following symbols:

$Y(t)$ = aggregate real disposable income in period t

$C(t)$ = real consumption in period t in the category of interest

$P(t)$ = population in time t

$y(t) = Y(t)/P(t)$ = per capita current income

$\bar{Y}P(t)$ = per capita permanent income

$c(t) = C(t)/P(t)$ = per capita consumption

$S(t)$ = per capita stock of durables

* The classic work of S. J. Prais and H. S. Houthakker, "The Analysis of Family Budgets, with an Application to two British Surveys Conducted in 1937-9 and Their Detailed Results:", Cambridge (Eng.) University Press, 1955, discusses the implications of various transformations on data for measured consumer behavior.

The most common sort of consumption function found in the literature rests on the partial adjustment assumption. Ordinarily, the assumption is made that there exists a target level of per capita consumption given by

$$(3.1) \quad c^*(t) = \alpha + \beta y(t).$$

Actual consumption is adjusted towards c^* according to

$$(3.2) \quad c(t) - c(t-1) = \gamma [c^*(t) - c(t-1)].$$

According to this formulation, consumption is smoothed and fluctuations in income have their main impact upon savings. If we substitute the first equation into the second, we obtain the form actually estimated:

$$(3.3) \quad c(t) = \alpha\gamma + \beta\gamma y(t) + (1-\gamma)c(t-1).$$

Equation (3.3) can also be derived as a consequence of a simple form of the Permanent Income hypothesis.* Suppose consumption is given by

$$(3.4) \quad c(t) = \alpha + \beta y_p(t),$$

where $y_p(t)$ is per capita permanent income in period t . This quantity is most often approximated by the following geometrically weighted sum of prior incomes:

$$(3.5) \quad \begin{aligned} y_p(t) &= \gamma \sum_{i=0}^{\infty} (1-\gamma)^i y(t-1-i) \\ &= \gamma y(t) + (1-\gamma)y_p(t-1). \end{aligned}$$

If we subtract the equation

$$(3.6) \quad (1-\gamma)c(t-1) = (1-\gamma)\alpha + (1-\gamma)\beta y_p(t-1)$$

* See Milton Friedman, "A Theory of the Consumption Function", A study by the National Bureau of Economic Research, New York. Princeton, Princeton University Press, 1957.

from the equation (3.4) for $c(t)$, we obtain

$$(3.7) \quad c(t) - (1-\gamma)c(t-1) = \alpha\gamma + \beta[yp(t) - (1-\gamma)yp(t-1)].$$

Using the identity relating $yp(t)$ and $yp(t-1)$ in (3.5), this becomes

$$(3.8) \quad c(t) = \alpha\gamma + \beta\gamma y(t) + (1-\gamma)c(t-1),$$

which is identical to (3.3).

It is worthwhile rewriting equation (3.3) in terms of aggregate quantities. Let us denote the estimate of $\alpha\gamma$ by b_0 , the estimate of $\beta\gamma$ by b_1 , and the estimate of $(1-\gamma)$ by b_2 :

$$(3.9) \quad C(t) = b_0 P(t) + b_1 Y(t) + b_2 \frac{C(t-1)P(t)}{P(t-1)}$$

The short-run effects of income and population are given by:

$$SRMPC = b_1 \quad SRMPE = b_0 + b_2 \frac{C(t-1)}{P(t-1)}.$$

(SRMPE is the short-run marginal population effect, the current quarter change in total consumption caused by adding one to the population.) If an increase in income or population is maintained long enough for consumption to stabilize, $C(t) = C(t-1)$ so that

$LRMPC = b_1/(1-b_2) = \quad$, $LRMPE = b_0/(1-b_2) = \quad$. Even though the short run dynamics differ, steady state properties are the same.*

* All of this is under the assumption of no secular growth in population or aggregate income. The same analysis of short and long-run effects can be carried out under the assumption of steady growth in population and/or aggregate income, but for reasonable growth rates the differences are trivial.

The mean lag in (3.8) is clearly $(1-\gamma)/\gamma$, from the last chapter. This is the mean lag of $c(t)$ behind $y(t)$; there is no corresponding estimate for $C(t)$ and $Y(t)$, as equation (3.9) is not a distributed lag relation in the usual sense.*

Note that population growth will almost always increase current aggregate consumption, but that it will have a positive long-run effect only if the estimated equation has a positive intercept.

In their recent book, Hendrik Houthakker and Lester Taylor (1966) have popularized a form of consumption equation originally developed by Gregory Chow (1957) and Marc Nerlove (1958). This model assumes that

$$(3.10) \quad c(t) = \alpha + \beta y(t) + \delta s(t).$$

For durables, the new variable $s(t)$ may be interpreted as the real per capita stock of consumer durables on hand at the start of period t . Houthakker and Taylor extend this approach beyond durable goods, however. They interpret $s(t)$ as a psychic "stock of habits" in the case of non-durables and services.

In either case, if we assume depreciation of the stock at a constant rate, μ , we have the identity

$$(3.11) \quad s(t) = c(t-1) + (1-\mu)s(t-1), \quad \text{or} \\ c(t-1) = s(t) - (1-\mu)s(t-1).$$

Lagging equation (3.10) and multiplying by $(1-\mu)$, we have

$$(3.12) \quad (1-\mu)c(t-1) = (1-\mu)\alpha + \beta(1-\mu)y(t-1) + \delta(1-\mu)s(t-1).$$

* If the rate of growth of population r is a constant, then the coefficient $b'_2 = b_2(1+r)$ on lagged consumption measures the aggregate lag distribution in the usual sense.

Subtracting (3.12) from (3.10), we obtain

$$(3.13) \quad c(t) - (1-\mu)c(t-1) = \alpha\mu + \beta[y(t) - y(t-1)] + \beta\mu y(t-1) \\ + \delta[s(t) - (1-\mu)s(t-1)].$$

Realizing that the last term in (3.13) is equal to $c(t-1)$ by the identity (3.11) above, the equation to be estimated becomes

$$(3.14) \quad c(t) = \alpha\mu + \beta[y(t) - y(t-1)] + \beta\mu y(t-1) + (1-\mu+\delta)c(t-1)$$

Let us denote the estimate of $\alpha\mu$ by b_0 , the estimate of β by b_1 , and so on. If we then write this equation in terms of aggregate variables, we have

$$(3.15) \quad C(t) = b_0 P(t) + b_1 Y(t) + (b_2 - b_1) \frac{Y(t-1)P(t)}{P(t-1)} \\ + b_3 \frac{C(t-1)P(t)}{P(t-1)}$$

The short-run effects on total consumption of unit changes in income and population are given by

$$SRMPC = b_1 \quad SRMPE = b_0 + (b_2 - b_1) \frac{Y(t-1)}{P(t-1)} + b_3 \frac{C(t-1)}{P(t-1)}$$

Depending on b_2 and b_1 , the short-run derivative with respect to population may be of either sign. If we set $C(t) = C(t-1)$, $P(t) = P(t-1)$, and $Y(t) = Y(t-1)$, we can examine the long-run derivatives and find

$$LRMPC = b_2 / (1 - b_3) = \frac{\beta\mu}{\mu - \delta} \quad LRMPE = b_0 / (1 - b_3) = \frac{\alpha\mu}{\mu - \delta}$$

An estimate of μ , the rate of depreciation of the stock, is given by b_2/b_1 . The parameter δ can be estimated from $[b_3 - 1 + b_2/b_1]$. Typically, however, we are interested mainly in the sign of δ . If δ is less than zero, there is, in the Houthakker-Taylor terminology, "satiation": the more stock on hand, the less additional stock is desired at any level of income. If on the other hand, δ is positive, we have "habit formation"; the higher past consumption has been, the greater is the desire for future consumption.

Houthakker and Taylor contend that satiation is likely to be observed for durables, while habit formation should be the rule for non-durables and services. If satiation prevails, we have a SRMPC above the LRMPC and an accelerator effect. This may become a bit clearer when we write the LRMPC in terms of the underlying parameters as

$$\text{LRMPC} = \frac{\beta\mu}{\mu - \delta}.$$

This is less than β , the SRMPC, if and only if δ is negative.

We can use the tool of the lag polynomial developed in the last chapter to find the mean lag of equation *3.14). In terms of estimated parameters, this equation may be written as

$$(3.14') \quad c(t) = \frac{b_0}{1 - b_3L} + \frac{b_1 - b_1L + b_2L}{1 - b_3L} y(t),$$

where L is the lag operator. If the coefficient of $y(t)$ is denoted by $G(L)$, the discussion of the last chapter should have made it clear that the mean lag

is just $G'(1)/G(1)$, or $(b_2 - b_1 + b_1 b_3)/(b_2(1 - b_3))$. In terms of the underlying parameters, the mean lag is simply $\delta/[\mu(\mu - \delta)]$. This mean lag rises with δ and falls with μ .

It must be recognized that these are only the most common of a whole host of alternative functional forms.* Both were tried for all categories of consumption, but it was not always the case that either one was superior to other possibilities.

The Equations

a. Aggregate Consumption Behavior

Before presenting our estimated behavioral equations, we define the following variables:

$$(3.16) \quad Y = (YDPI/PCPTOT)(1000.0/LTPOP).$$

$$(3.17) \quad CTOT = CPTOT.LTPOP$$

The quantity Y is thus per-capita disposable income in constant dollars; it will be used as the $y(t)$ of the last section. An aggregate per capita real consumption function specified according to Houthakker-Taylor yielded the following estimates:

$$(3.18) \quad CPTOT = 25.07 + .7820 CPTOT(t-1) + .1884 Y(t-1) + .6868 \Delta Y$$

(1.53)
(8.24)
(2.26)
(7.14)

$$R^2 = .997$$

$$SE = 10.74$$

$$DW = 2.27$$

* The influence of non-human wealth on the consumption function has been persuasively argued by Franco Modigliani and co-authors. A good summary reference will be found in Franco Modigliani and Albert Ando, "Econometric Analysis of Stabilization Policies", The American Economic Review, Vol. LIX, No. 2, May 1969.

The standard error translated into dollar totals is \$1.88 billion, which is obtained by multiplying the per capita standard error of estimate by average population. The short-run marginal propensity to consume is .68 (obtained directly from the last term in Equation (3.18) while the long-run marginal propensity to consume of .85 equals the average propensity, as one would expect where the average propensity has been stable.* Since the SRMPC is less than the LRMPC, equation (3.18) indicates net habit-formation in the Houthakker-Taylor terminology. The estimated depreciation rate of the (mostly psychic) stock is 27.4% per quarter, and the mean lag is approximately one quarter. The short-run marginal population effect is \$311, while the LRMPE is \$115. Both figures are small relative to the sample average of CPTOT, \$1816.

The low standard error compares favorably with other similar relations, although as we shall see, the simulated standard error is higher. This is inevitable in a properly specified equation where lagged dependent variables appear, as they do throughout this study.

b. Separate Equations

We also present four individual equations for durables, non-durables, housing services and non-housing services. Variants on this breakdown appear in many standard econometric models (automobiles sometimes are explained separately from other durables, while consumer services are often not separated as we felt obliged to do) and so are of interest for comparative purposes. Second, proper aggregation levels remain an unresolved issue in econometric model building so that comparisons with the total equation is worthwhile. Third, it will turn out that the dynamics are enough different among these major categories, so that for some dynamic purposes only the separate equations should be used. Fourth, certain fiscal policies are specific to a given sector, which the disaggregated version allows to be treated explicitly.

* This is absolutely correct when the intercept term is zero; when a small intercept (very small relative to the average of the dependent variable) appears as in (3.18), the statement is approximate.

The first identity in this sector relates the per-capita quantities to total deflated consumption:

$$(3.19) \quad CTOT = (LTPOP/1000.0) * (CND + CDUR + CNS + CHS).$$

We now consider the four separate behavioral equations of this sector, all estimated over the period 1954I - 1967IV.

The Houthakker-Taylor form worked best for non-durables:

$$(3.20) \quad CND = 136.18 + .2447 [Y - Y(t-1)] + .1184 Y(t-1) + .5636 CND(t-1)$$

(3.49) (5.32) (3.23) (4.34)

$$R^2 = .992$$

$$SE = 5.27$$

$$DW = 2.11$$

Figures in parentheses beneath the coefficients are the absolute values of the associated t-statistics. The quantity labeled SE is the standard error of the equation, and the rest of the relation should be self-explanatory. The estimate of δ is .049, indicating mild habit-formation. Since the stock here is a psychic stock, the estimated depreciation rate of 48.5% per quarter is not unreasonable. Using the formulae developed in the last section, we can compute the following quantities from (3.19):

$$SRMPC = .245 \qquad SRMPE = 364.$$

$$LRMPC = .272 \qquad LRMPE = 312.$$

All these are evaluated at the point of sample means. Both the SRMPC and LRMPC seem reasonable. The two population effects are less than half the mean of CND over the period. The mean lag of equation (3.19) is short: .82 quarters, but in accordance with casual observation of consumer behavior.

The standard Houthakker-Taylor form also fitted the durables data well, but a somewhat better equation, in terms of goodness of fit and theoretical parameter values, was found by generalizing the lag structure slightly: *

$$\begin{aligned}
 (3.21) \quad \text{CDUR} &= .2666 [\Delta Y] + .2610 [\Delta Y(t-1)] + .0200 Y(t-2) \\
 &\quad (3.59) \quad (3.28) \quad (2.90) \\
 &+ .8416 \text{CDUR}(t-1) \\
 &\quad (16.5)
 \end{aligned}
 \qquad
 \begin{aligned}
 R^2 &= .975 \\
 SE &= 7.99 \\
 DW &= 2.36
 \end{aligned}$$

The statistically insignificant intercept was dropped from the equation. We cannot compute the structural parameters of the Houthakker-Taylor model here, but we can calculate the short-run and long-run income and population effects at the point of means as follows:

$$\begin{aligned}
 \text{SRMPC} &= .267 & \text{SRMPE} &= -175 \\
 \text{LRMPC} &= .126 & \text{LRMPE} &= 0.0
 \end{aligned}$$

Since the LRMPC is above the SRMPC, it is clear that the equation indicates a satiation mechanism, rather than habit-formation. This, of course, is just what we would expect to observe with durables. The negative SRMPE is about 64% of the sample mean of CDUR. Because the LRMPC is below the SRMPC, some of the lag coefficients (the w_i in the notation of Chapter II) are negative. In this case there is no obvious measure of the speed of adjustment.

At the outset we tried a single equation to explain total services consumption per capita. As with most other related equations, we obtained

* There is a larger transient effect of income and more reasonable steady state values in this equation, in contrast to the Houthakker-Taylor form.

equations in which the lagged dependent variable had a coefficient of approximately unity and was contributing most of the explanatory power, because services consumption is highly correlated with time and hence its own lagged value. This empirical result is not consistent with the interpretation of the coefficient of the lagged dependent variable as one minus a speed of adjustment. Total services were split into CHS and CNS to find out whether the result was due to aggregation difficulties. Housing services were, in fact, almost entirely trend, but non-housing services responded noticeably and sensibly to changes in income.*

The best equation for non-housing services was of the simple partial adjustment permanent income form:

$$(3.22) \quad \text{CNS} = 33.389 + .0371 Y + .7549 \text{CNS}(t-1)$$

(2.28) (3.16) (8.74)

$R^2 = .989$
 $SE = 3.06$
 $DW = 1.97$

The lagged dependent variable is quite important here, but the mean lag of 3.08 quarters is much less than for total services. The short-run and long-run derivatives evaluated at the sample means are as follows:

$$\begin{aligned} \text{SRMPC} &= .037 & \text{SRMPE} &= 356. \\ \text{LRMPC} &= .151 & \text{LRMPE} &= 136. \end{aligned}$$

Both the short-run and long-run population effects are well below the sample mean of CNS, which was 430.04.

* The reader should read the Department of Commerce Publication, The National Income and Product Accounts of the United States, 1929-1965. A supplement to the Survey of Current Business, U.S. Dept. of Commerce, Office of Business Economics, August 1966, for a full understanding of how this (and other series) are constructed. Partly for practical reasons and partly for theoretical reasons, HS is almost more trend.

As mentioned above, non-housing services proved to be almost entirely trend. No equation with a lagged dependent variable had a significant income term or a significant intercept. A large part of this category consists of imputed rent on owner-occupied houses, but the lack of an income effect is still surprising, especially since we are considering per-capita expenditure.* The following was the most sensible equation we could devise for this category:

$$(3.23) \text{ CHS/Y} = .0056 + .9582 [\text{CHS}(t-1)/\text{Y}(t-1)]$$

(1.66) (36.4)

$$\begin{aligned} R^2 &= .961 \\ SE &= .0011 \\ \bar{Y}.SE &= 2.16 \\ DW &= 1.47 \end{aligned}$$

The implied mean lag of this equation is an absurd 22.8 quarters. No population effects are indicated. The short-run MPC is .128, while the LRMP is .132; both are reasonable. The Durbin-Watson is moderately low, and the intercept is not significant. Clearly this equation is far from satisfactory (on theoretical and statistical grounds and cannot be expected to perform well beyond the period of fit, but it was the best we could find. Removing the intercept, for instance, gave the lagged variable a coefficient in excess of unity. This means that HS/Y will rise without limit - an absurd result.

When the single aggregate equation and aggregation of the four individual equations are compared, the single equation version clearly dominates. Its standard error of \$1.88 billion compares to \$3.22 billion

* This troublesome category was taken as exogenous to "A Quarterly Econometric Model of the United States: A Progress Report", by Maurice Libenberg, et al, Survey of Current Business, May, 1966, Vol.46, No.5, and this course is certainly attractive. Yet, since HS is logically endogenous, we felt compelled to explain it within the model.

for the sum of the separate standard errors.* More important, in terms of structural interpretation, is that, while the short-run marginal propensity to consume of (3.18) is approximately equal to the total of the SRMPC's of (3.20)-(3.23), the total long-run MPC for the individual equations is only .68. This equals the short-run MPC and is suspiciously far below the average propensity to consume. Hence, we shall restrict ourselves to the aggregate equation when the entire model is assembled. Perhaps, the differential performance is explained by leaving out relative prices in the individual equations. These excluded substitution effects among consumption components are automatically subsumed by the aggregate equation.

Simulations

The consumption sector consists of the total per capita equation (3.18) and identity (3.17) or the identity (3.19) and the behavioral equations (3.20-3.23); the identities merely serves to simplify presentation. This sector was dynamically simulated over the sample period, 1951-1967IV and over the four quarters of 1968. The results are summarized in Table III.2. It is most interesting to compare error performance in \$ billions, for that is what finally appears in a GNP model.

During the sample period, the total per capita equation has a RMS of \$3.14 billions, while the summed individual error was substantially larger at \$5.24. Whereas prediction errors in 1968 were substantially larger for both methods of forecasting total consumption, the mean error for the total equation was about half that for the individual equations. One big advantage

*

The latter amount is obtained by summing the individual standard errors of estimate and multiplying by average population. This ignores error term covariances, but as will be immediately apparent in the simulations which follow, neglecting them does not make any difference in the comparisons of interest.

of the individual equations is that it permits the investigator to isolate the error source; it is clear that CDUR contained just about all prediction error during 1968. Thus, on the basis of simulation behavior and theoretical properties, we have chosen the total per capita equation, even though long-run econometric progress will come most surely from building correct models of individual sectors.

Turning now to simulations of individual equations within the sample period, the weakest equation in terms of relative RMS error is the one for CDUR. But since durables are the most volatile category of consumption, this is no great surprise. The housing services equation goes off track when simulated over the period, and it significantly under-predicts that category of consumption. Consequently, the predictions of total consumption are biased downward also.

In 1968, the non-durables equation works well, and the housing services equation does also. This latter result seems to indicate that the CHS equation can be expected to predict well for short simulations, but, as we saw above, it will go off the track if allowed to generate its own lagged values for long periods. The durables equation is much worse in 1968 than in the sample period; the relative RMS error is larger and there is evidence of systematic under-prediction. This would seem to indicate a problem of specification. The non-housing services equation still has a low RMS error, but its predictions appear to be biased upwards.

Table III.2
Consumption Sector Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
(1954I - 1967IV)					
Total Consumption Equation					
CPTOT (per Capita/year)	1815.61	17.27	.95	- .72	- .309
CPTOT (\$ billions/year)	330.00	3.14	.95	- .20	- .47
Individual Consumption Equations					
CND (per Capita/year)	848.72	5.99	.76	- .53	- .67
CDUR (per Capita/year)	273.85	10.93	3.98	1.67	1.18
CNS (per Capita/year)	430.04	4.34	1.00	.33	.59
CHS (per Capita/year)	275.10	5.19	2.02	-1.73	-2.66**
CTOT (\$ billions/year)	331.99	5.24	1.58	-1.39	-2.07**
Average Population (mills.)	181.79	-	-	-	-
(1968I - 1968IV)					
Total Consumption Equation					
CPTOT (per Capita/year)	2289.16	63.40	2.77	-63.15	-19.45***
CPTOT (\$ billions/year)	460.73	12.76	2.77	-12.71	-19.45***
Individual Consumption Equations					
CND (per Capita/year)	978.43	7.07	.72	- 1.33	- .34
CDUR (per Capita/year)	401.06	35.21	8.78	-32.92	- 4.42**
CNS (per capita/year)	496.00	4.79	.96	4.47	4.40**
CHS (per Capita/year)	331.48	3.02	.91	- 1.26	.78
CTOT (\$ Billions/year)	460.78	22.82	4.95	-22.79	-36.21***
Average Population (mills.)	201.26	-	-	-	-

** Significant at 5%

*** Significant at 1%

CHAPTER III; Personal Consumption Expenditure

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Implications of Data Transformation

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(This provides more information than the 1965 Supplement on how the series were constructed.)

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CHAPTER IV

Fixed Investment Spending

The decision to invest in long-lived capital assets will depend on expectations about the future. The formation of such expectations is exceedingly hard to model; different mechanisms may generate expectations at different times. As a consequence, neither this nor any other econometric model of investment behavior is particularly good at characterizing fixed investment spending. Despite efforts to maintain simplicity throughout, the inherent complexity in both the concepts and dynamic properties of investment cause this chapter to be the most complicated in this volume.

Our approach to the investment sector contains some novelty. We obtain external estimates of the distributed lags between the decision to invest and investment spending and impose these lags on our behavioral equations. Hence, statistical estimation is focused directly on the process of expectation formation. Our approach is in the spirit of a recent paper of Charles Bischoff (1970), though we have extended his formulation in some respects. Most recent work on the investment decision derives in one way or another from Jorgenson's studies (1963), and ours is no exception.

The variables employed in this sector are described below. We then explain the basic model underlying our investigation. Chapter II on distributed lags should be reviewed before reading this theoretical discussion. The estimated equations are presented, and the chapter concludes with simulation

experiments. An appendix to this chapter shows the derivation of the estimated order-delivery lag for producers' durable equipment as well as the construction of the user cost variables.

THE DATA

The data series for this sector have been listed in Table IV.1. The first four investment variables are seasonally-adjusted quarterly totals expressed at annual rates and were taken from the Survey of Current Business. CDSTS is defined as the number of private non-farm housing starts, in billions of units at annual rates, times the average value per start expressed in dollars, deflated by the implicit deflator for private non-farm residential construction. A variety of apparently identical series on housing starts and cost per unit are published by government agencies. The only starts series that gave consistent results with National Accounts data was taken from the Bureau of Labor Statistics Bulletin 1260 for 1953-1958, from 1965-1968 from the Bureau of the Census Series C-20, and for the intervening years from unpublished data sent by the Bureau of the Census. Quarterly averages of seasonally-adjusted monthly totals at annual rates were used. The single usable cost series was taken from the Historical Appendix to the Census Series C-20 for 1953-1958, while the unpublished data for later years was furnished by the Construction Statistics Division of the Bureau of the Census. Quarterly averages of monthly figures were used. Finally, the implicit deflator was taken from the Census publication Housing Statistics.*

* We feel that researchers in this area should not have to go through the exhausting process of trial and error necessary to find the single CDSTS series that moves with INVH. The series used by the Commerce Department to construct INVH should be published and indicated as such.

TABLE IV.1Variables Appearing in the Investment Sector

(Endogenous Variables - Determined in Investment Sector)

INFXD	Gross private domestic fixed investment, billions of constant dollars.
INVEQ	Gross private domestic investment in producers' durable equipment, billions of constant dollars.
INPL	Gross private domestic investment in non-residential structures, billions of constant dollars.
INVH	Gross private domestic investment in residential structures, billions of constant dollars.
CDSTS	New private non-farm housing starts, billions of constant dollars.
STKEQ	Net stock of producers' durable equipment, end of quarter, billions of constant dollars.
STKPL	Net stock of non-residential structures, end of quarter, billions of constant dollars.
STKH	Net stock of residential structures, end of quarter, billions of constant dollars.
IN90	Smoothed rate of inflation, percent per year (defined in text).
UCPI	User cost of non-residential structures, including the impact of inflation, percent (defined in Chapter IV appendix).

(Endogenous Variables - Determined Elsewhere in the Model)

RCB	Yield on AAA-rated corporate bonds, percent.
RCP	Yield on 4-6 month prime commercial paper, percent.
YGPP	Gross private product, billions of constant dollars.
YDPI	Disposable personal income, billions of current dollars.
PCPTOT	Implicit price deflator for total personal consumption expenditures, 1957-59 = 1.00.
PGPP	Implicit price deflator for gross private product, 1957-59 = 100.

(Exogenous Variables)

DQ2	Seasonal dummy variable, equal to 1.0 only in second quarters, zero otherwise.
DQ3	Seasonal dummy variable, equal to 1.0 only in third quarters, zero otherwise.
DQ4	Seasonal dummy variable, equal to 1.0 only in fourth quarters, zero otherwise.
LTPOP	Total population, billions of persons.

(Externally determined Parameters)

WDP	Weight of deterioration rate in non-residential structures user cost, pure number.
WRP	Weight of discount rate in non-residential structures user cost, pure number.
WIP	Weight of rate of inflation in non-residential structures user cost, pure number.

Note: All flow variables from the National Income Accounts are seasonally adjusted quarterly totals measured at annual rates.

Values for the three stocks at the end of 1946 were taken from Raymond W. Goldsmith (1962).^{*} Values for later quarters were computed according to the following identities:

$$(4.1) \quad \text{STKEQ} = (1/4) \text{INVEQ} + (1 - .148/4) \text{STKEQ}(t-1)$$

$$(4.2) \quad \text{STKPL} = (1/4) \text{INPL} + (1 - .0586/4) \text{STKPL}(t-1)$$

$$(4.3) \quad \text{STKH} = (1/4) \text{INVH} + (1 - .024/4) \text{STKH}(t-1)$$

We divide investment at annual rates by four to obtain the quarterly total. The coefficients of the lagged stocks are equal to one minus the quarterly rate of deterioration. We assume, as the equations show, that equipment deteriorates at 14.8% per year and that plant deteriorates at 5.86% per year. For the derivation of these estimates, see the appendix to this chapter. The estimated deterioration rate of the housing stock, 2.4% per annum, was taken from the M.I.T.-Federal Reserve Board-University of Pennsylvania model (de Leeuw and Gramlich, 1968).^{**}

The exponentially smoothed rate of inflation, IN90, is computed according to

$$(4.4) \quad \text{IN90} = (4)(100)[(\text{PGPP} - \text{PGPP}(t-1))/\text{PGPP}(t-1)](.10) + (.90)\text{IN90}(t-1).$$

* Goldsmith (1962) constant dollar stock estimates at 1946 year end were multiplied by the implicit deflator (obtained by dividing current dollar 1958 stocks by constant dollar 1958 stocks denominated in 1947-49 prices in order to convert them into constant 1958 dollars. For each of the three capital series, all sectors except the two government sectors were included.

The factor of four converts the first term to an annual rate of inflation, the factor of 100 makes it a percentage, and the next expression is self-explanatory. The weights of .10 and .90 on current inflation and lagged IN90 indicate that IN90 changes slowly. This is in accord with most studies of the formation of inflationary expectation in times of fairly stable prices and IN90 is used as a measure of expected inflation in this model.* The starting value for IN90 at the end of 1946 was obtained by smoothing annual changes in PGPP starting in 1929.

UCPI and the other user costs are explained in the appendix. Four user costs were employed in our investigations. These were computed as follows:

$$(4.5) \quad UCPI = 5.86 \text{ WDP} + (2.55 \text{ RCB}) \text{ WRP} - \text{IN90 WIP}$$

$$(4.6) \quad UCP = 5.86 \text{ WDP} + (2.55 \text{ RCB}) \text{ WRP}$$

$$(4.7) \quad UCEI = 14.8 \text{ WDE} + (2.55 \text{ RCB}) \text{ WRE} - \text{IN90 WIE}$$

$$(4.8) \quad UCE = 14.8 \text{ WDE} + (2.55 \text{ RCB}) \text{ WRE}$$

The quantity (2.55,RCB) is taken as the relevant rate of discount. As explained more fully in the appendix, the 2.55 multiple of the corporate bond rate reflects the higher cost of equity capital and prevalent debt-equity ratios. Only UCPI is used in the final sector, as we shall see below.

The first two endogenous variables determined elsewhere are private interest rates obtained from quarterly averages of monthly figures taken from the Federal Reserve Bulletin. The next two variables are standard income and

*See Feldstein and Eckstein (1970), for instance, and the literature there cited.

and product figures from the Survey of Current Business, and the last two are also from the Survey. LTPOP is the quarterly average of monthly figures from the Survey.

The three externally determined parameters are the weights used in computing UCPI. They are explained in the appendix and depend on government tax policy.

Basic Theoretical Framework

Two basic ideas underly most discussions of investment spending. These are the accelerator - the relation between output and the capital stock - and the cost of capital. We shall discuss these concepts as they relate to business investment in plant and equipment. Allowing for lags in behavior, our model of the investment process will be constructed from these key concepts. At the end of this section, we shall consider housing investment.

Suppose production takes place under constant returns to scale. Then for any set of prices and interest rates, the capital-output ratio that minimizes costs is independent of the level of output. Let Y be business output and let KSI be the cost-minimizing (instantaneously optimal) capital stock. If all prices and interest rates are constant, we can write.

$$(4.9) \quad KSI = v Y,$$

where v is the cost-minimizing capital-output ratio.

Net investment is defined as the change in capital stock, while gross investment is the change in the capital stock plus spending to replace worn-out units of the stock. (In the national accounts, net investment is gross investment minus depreciation.) If all prices and interest rates are

constant and if actual capital stock is always equal to desired capital stock (KSI), the level of net investment will be proportional to the change in output. This is the simplest version of the accelerator; it serves as the basis for a number of simple analytical business cycle models.*

It is easy to incorporate the cost of capital into this model. The more expensive it is to have funds tied up in plant and equipment, the fewer investment projects it will be profitable to undertake. The rate of interest, a measure of the cost of funds, affects the cost of capital. Cost of capital is a complex magnitude, subject to influences from debt-equity mixture, tax laws, expected inflation, and the durability of the capital stock. The Appendix contains a careful derivation. Denote the cost of capital by r . Then the foregoing discussion implies that v in equation (4.9) is a declining function of r , so we can write

$$(4.9') \quad KSI = v(r) Y, \text{ with } dv/dr < 0.$$

The augmented acceleration principle still asserts that if actual capital stock is always equal to desired capital stock, the level of net investment will equal the change in KSI.

A number of modifications must be made to this basic model.**

First, it is desirable to formulate investment decisions in terms of gross

* See, for instance, Paul A. Samuelson, (1939), and R.G.D. Allen (1956), Chapter 3.

** See Knox (1952).

investment, since reported depreciation figures may not measure actual deterioration of capital very well. Let I be gross investment spending, and assume that due to deterioration a fraction g of the capital stock is lost every period and must be replaced. Then gross investment will be given by

$$(4.10) \quad I(t) = KSI(t) - KSI(t-1) + gK(t-1),$$

where K is the actual (end of period) capital stock.

A more serious problem with this rudimentary accelerator formulation is that it does not adequately consider the amount of capital on hand. Suppose the economy is in a deep depression, with substantial amounts of redundant labor and capital. In this situation, KSI may be well below K , so that a rise in KSI may not affect investment at all. Allowing for deterioration, the amount of capital available during the current period if no gross investment is undertaken will be $(1-g)K(t-1)$. Desired gross investment will equal the difference between this carry-over stock and the current value of KSI . Actual gross investment will equal desired gross investment unless the latter is negative. Since this is likely to occur only in very severe depressions, we shall replace (4.10) by

$$(4.11) \quad I(t) = KSI(t) - (1-g)K(t-1).$$

We must now consider the incorporation of lags in the investment process into our model.* Two sorts of lags really matter. First, there is a lag between changes in the determinants of KSI and business decisions

*

The reader should be certain he understands Chapter II before proceeding.

to purchase capital goods. The second lag is the time between the decision to invest and actual investment spending; this is most sensibly thought of as the lag between orders and deliveries of investment goods. We shall analyze these two lags in turn.

KSI is that capital stock which would maximize profits given current Y and r . But capital goods will be used in the future as well as in the current period. The fact that sales are high this month will lead to business investment only if sales are expected to be high in the future as well. Here, as in the last chapter, it is assumed that expectations of the future are based on past experience. Thus the stock actually preferred, which we shall call KS , will depend on current and past values of KSI. The relation between these two quantities is assumed to be given by

$$(4.12) \quad KS(t) = P(L) \text{ KSI}(t),$$

where $P(L)$ is a lag polynomial reflecting expectation formation and lags in decision-making. KSI may be viewed as the true optimal stock plus independent random disturbances, so that a lag polynomial such as $P(L)$ serves to reduce the variability of the capital budgeting process by acting as a smoothing or averaging device.

Replacing KSI in (4.11) by KS , we obtain

$$(4.13) \quad I(t) = P(L) \text{ KSI}(t) - (1-g)K(t-1).$$

We shall work with estimates of g that have been relegated to the Appendix. Then, since I and K are both observable series, for estimation

purposes we would probably work with (4.13) in the form

$$(4.13') \quad I(t) + (1-g)K(t-1) = P(L) v(r) Y,$$

making use of (4.9'). The dependent variable could be calculated given an independently obtained estimate of g and alternative forms of $P(L)$ and $v(r)$ could be estimated, tested and compared.

But we are not quite ready to discuss estimation, as the order-delivery lag has yet to be reckoned with. Let $S(t)$ be investment "starts" in period t . In the case of business investment, it makes most sense to think of $S(t)$ as new orders placed in period t . As it takes some time to fill orders for capital goods, some deliveries of capital will be made in current and future periods as a result of past starts. These deliveries must be taken into account when making decisions on $S(t)$.

The discrepancy between desired and carry-over capital stock is simply $[KS(t) - (1-g)K(t-1)]$, as discussed above. Current starts will be set equal to this discrepancy minus the backlog of capital to be delivered as a result of past starts; this backlog is simply the sum of all past starts minus all past completions (expenditures). Adding current starts, the quantity to be set equal to the discrepancy between desired and carry-over stock is

$$\begin{aligned} (4.14) \quad & S(t) + S(t-1) + S(t-2) + \dots - I(t-1) - I(t-2) - \dots \\ &= (1 + L + L^2 + L^3 + \dots) [S(t) - I(t-1)] \\ &= \frac{1}{1-L} [S(t) - I(t-1)], \end{aligned}$$

where the geometric series in L has been summed. Setting this quantity equal to the discrepancy between desired and carry-over capital stock and multiplying through by $(1-L)$, we obtain

$$(4.15) \quad S(t) = [KS(t) - KS(t-1)] + I(t-1) - (1-g)K(t-1) + (1-g)K(t-2).$$

This equation can be simplified by using the identity relating gross investment to changes in the capital stock,

$$(4.16) \quad K(t) = I(t) + (1-g)K(t-1),$$

to eliminate the $(1-g)K(t-2)$ term. This yields the final equation for investment starts:

$$(4.17) \quad \begin{aligned} S(t) - gK(t-1) &= [KS(t) - KS(t-1)] \\ &= P(L) [KSI(t) - KSI(t-1)]. \end{aligned}$$

must be made;

One final step, an operational expression for starts is required in order to obtain an equation useable for explaining gross investment. To obtain it, investment will be written as a distributed lag function of current and past starts. Thus:

$$(4.18) \quad I(t) = Q(L) S(t), \text{ or } S(t) = [1/Q(L)] I(t),$$

where $Q(L)$ is a known lag polynomial.* If all starts were finished (delivered) in one period, we would have $Q(L) = 1$. If three quarters are delivered in one period and one quarter in two periods, $Q(L)$ would equal $.75 + .25L$.

*The idea of imposing a known $Q(L)$ in this context originates with Charles Bischoff (1970).

Substituting the expression for $S(t)$ from (4.18) into (4.17) and multiplying through by $Q(L)$, we obtain the desired equation:

$$(4.19) \quad I(t) - gQ(L)K(t-1) = P(L)[Q(L)(KSI(t)-KSI(t-1))].$$

Let us now outline how equations (4.18) and (4.19) will be employed in what follows. In the case of housing, the series CDSTS corresponds fairly closely to the theoretical series $S(t)$. An equation like (4.18) is used to explain housing investment, $INVH$, given CDSTS.

The instantaneously desired stock of housing should depend on per-capita income, the population, and the cost of home-owning. We specify the form of the function involved and estimate its parameters along with those of $P(L)$ using (4.17).

For plant and for equipment, equation (4.19) is used. The dependent variable is computed from the $I(t)$ and $K(t)$ series, the known constant g , and the known lag polynomial $Q(L)$. We specify the form of $v(r)$ - see (4.9') - and estimate the parameters of this function first, giving a predicted value of $KSI(t)$, and then proceed to estimate the lag polynomial $P(L)$ second.

In all three cases, KSI is assumed to be a non-linear function of its determinants. Hence, simple non-linear estimation is employed, as we shall discuss below.

THE EQUATIONS

a. Housing

Let us begin with housing. It is not the case that CDSTS reflects all housing investment starts, but the correspondence is close. In 1960 and 1961, the mid-years of our sample, $INVH$ averaged \$21.75 billion.

Of this amount, \$.60 billion was for new farm construction and \$.45 billion was for the repair and renovation of existing structures. Thus, \$20.70 billion of the total represents new private non-farm construction, which corresponds to CDSTS.

The lag structure used by the Commerce Department to go from starts to completions in their calculations of INVH is the following (Sherman Maisel, 1965.)

$$(4.20) \quad QH(L) = .41 + .49L + .10L^2$$

The mean lag is about seven tenths of a quarter. Imposing this lag structure and using deflated disposable income and seasonal dummy variables to explain those components of INVH other than new private non-farm construction, we obtained:

$$(4.21) \quad \begin{array}{l} \text{INVH} = 4.2211 + 1.0721 [.41 \text{ CDSTS} + .49 \text{ CDSTS (t-1)} + .10 \text{ CDSTS (T-2)}] \\ \quad \quad \quad (6.10) \quad (32.56) \\ \quad + .0021 (\text{YDPI/PCPTOT}) - .2135 \text{ DQ2} - .4880 \text{ DQ3} - .3653 \text{ DQ4} \\ \quad \quad \quad (2.13) \quad \quad \quad (1.26) \quad \quad (2.87) \quad \quad (2.15) \end{array}$$

$$\begin{array}{l} R^2 = .955 \\ SE = .448 \\ DW = .886 \end{array}$$

This equation fits well, but the Durbin-Watson indicates high serial correlation in the residuals and the coefficient of the CDSTS term is a bit high. No variations on this specification performed better, though several were tried.

It is now necessary to explain CDSTS. We assumed that the instantaneously optimal housing stock, KSIH, on a per capita basis depends on per capita

real disposable income and the cost of capital:

$$(4.22) \quad \frac{KSIH}{LTPOP} = c[(YDPI/PCPTOT)/(LTPOP)]^a RCP^{-b}.$$

The constants a and $-b$ are clearly the elasticities of $KSIH$ with respect to real per capita disposable income and the commercial paper rate, respectively. The third constant, c , is merely a scaling factor. We follow Sherman Maisel (1965) in using the commercial paper rate rather than the more sluggish mortgage rate. When mortgage money is scarce the mortgage rate does not vary even though many potential borrowers are unable to find funds. The more volatile commercial paper rate captures the impact of non-price credit rationing more completely when the money markets are tight.

The estimated equations were variants of (4.17). This equation has to be multiplied by four, since $CDSTS$ is expressed at annual rates. A variety of assumptions were made about the form of $P(L)$, and a search was made to find that combination of the parameters a and b that minimized the sum of squared errors.* We found that the value of a , so long as it was near unity, did not affect the sum of squared errors much. We, therefore, assumed a unit income elasticity by setting a equal to one. The sum of squared errors was sensitive to the values of b , however. The best estimate was obtained with $b = .30$, although values of $.25$ and $.35$ were almost as good statistically. Writing

*

Under appropriate assumptions, which do not hold here, this is a maximum-likelihood method. More generally, this is an extension of least squares to a non-linear expression.

$$(4.23) \quad KSIH' = (YDPI/PCPTOT)/RCP^{.30}, \text{ and}$$

$$(4.24) \quad DVH = CDSTS - .024 STKH (t-1)$$

our equation for CDSTS may be written as

$$(4.25) \quad DVH = \underset{(2.29)}{.0368[KSIH'(t-1) - KSIH'(t-2)]} + \underset{(2.29)}{.0369[KSIH'(t-2) - KSIH'(t-3)]} \\ + \underset{(43.33)}{.9697 DVH(t-1)} \quad \begin{array}{l} R^2 = .809 \\ SE = 1.07 \\ DW = 1.86 \end{array}$$

Everything in this equation is reasonable except the mean lag of about 25 years. Within the framework used in this sector, no remedy for this is obvious.

b. Non-Residential Structures

If we assume that production functions have constant returns to scale, doubling output will double the instantaneously optimal capital stock. Were we further to assume that the aggregate production function is Cobb-Douglas, it is easy to show that doubling the user cost would divide the capital stock in half. Under other assumptions about the form of the production function, different elasticities of capital with respect to output and user cost will be obtained. In this study, we shall assume constant returns to scale, but we shall not fix the elasticity of capital demand with respect to user cost. We thus write the instantaneously optimal stock of non-residential structures as either

$$(4.26) \quad \text{KSIP} = cYGGP/UCP^a, \text{ or}$$

$$(4.27) \quad \text{KSIP} = cYGGP/UCPI^a$$

depending upon which user cost is employed. Here c is a scaling constant, and a is the elasticity of capital demand with respect to user cost. Multiplying (4.15) by four, since INPL is expressed at annual rates, we experimented with both (4.26) and (4.27), searching for the best estimate of the user-cost elasticity a in both cases.

For the lag structure governing the time delays between starts and completions of non-residential structures, we followed Bischoff (1970) and used the structure estimated by Mayer (1958):

$$(4.28) \quad QP(L) = .30 L + .38 L^2 + .18 L^3 + .11 L^4 + .03 L^5.$$

The mean lag here is 2.2 quarters.

Various forms of $P(L)$ were examined. The user cost without an inflation factor performed marginally better than the user cost that contained IN90. With $a = 1$, coefficient estimates were usually absurd. Much better fits were obtained with $a = 0$. The best estimate of a was .20. Even though this did not represent a statistically significant improvement over $a = 0$, it was decided to retain this estimate because of a strong a priori feeling that investment is influenced to some extent by changes in user cost.

Our best estimate of $P(L)$ was of the simple Koyck form. If we write

$$(4.29) \quad KSIP' = YGPP/UCP^{.20}, \text{ and}$$

$$(4.30) \quad DVP - INPL = .0586 QP(L) STKPL(t-1),$$

we can write our best equation for this category as

$$(4.31) \quad DVP = \underset{(3.40)}{.1195} [.30\Delta KSIP'(t-1) + .38\Delta KSIP'(t-2) + .18\Delta KSIP'(t-3)$$

$$+ .11\Delta KSIP'(t-4) + .03\Delta KSIP'(t-5)] + \underset{(54.66)}{.9607} DVP(t-1)$$

$$\begin{aligned} R^2 &= .906 \\ SE &= .557 \\ DW &= 2.06 \end{aligned}$$

The mean lag in (4.26) is 11.5 quarters. When this is added to the mean lag of $QP(L)$, we have the result that the mean lag between changes in the determinants of $KSIP$ and actual investment spending on plant is 13.7 quarters, which seems reasonable.

c. Producers Durable Equipment

The same basic approach was followed for investment in producers' durable equipment. We experimented with

$$(4.32) \quad KSIE = cYGPP/UCI^a, \text{ and}$$

$$(4.33) \quad KSIE = cYGPP/UCI^a.$$

With a variety of forms of $P(L)$, the estimate of a that consistently explained the data best was $a = 0$. Again, the Cobb-Douglas assumption $a = 1$ usually gave absurd results.

We estimated the lag structure between orders and spending for producers' durable equipment using monthly data for the machinery and equipment industries. The procedures and data sources are described in the Appendix to this chapter. Our estimate is the following, which has a mean lag of 2.3 quarters:

$$(4.34) \quad QE(L) = .319 + .266 L + .102 L^2 + .032 L^3 + .028 L^4 \\ + .054 L^5 + .082 L^6 + .083 L^7 + .033 L^8.$$

Again, we multiplied equation (4.15) by four since INVEQ is measured at annual rates. If we write

$$(4.35) \quad DVE = INVEQ - .148 QE(L) STKEQ(t-1),$$

our best equation for durable equipment spending can be written as:

$$(4.36) \quad DVE = .2942[.319\Delta YGPP + .266\Delta YGPP(t-1) + .102\Delta YGPP(t-2) \\ (7.71) \\ + .032\Delta YGPP(t-3) + .028\Delta YGPP(t-4) + .054\Delta YGPP(t-5) \\ + .082\Delta YGPP(t-6) + .083\Delta YGPP(t-7) + .035\Delta YGPP(t-8)] \\ + .9004 DVE(t-1) \\ (47.28) \quad R^2 = .981 \\ SE = .769 \\ DW = 1.857$$

The mean lag of $P(L)$ as estimated by (4.32) is about 9.2 quarters. Adding the mean lag of $QE(L)$, we have a total mean lag in producers' durable equipment spending of 11.5 quarters. This is shorter than was estimated above for non-residential structures, as one would expect.

The final equation of this sector is the identity that gives total constant-dollar fixed investment spending as the sum of its components:

$$(4.37) \quad \text{INFXD} = \text{INVH} + \text{INPL} + \text{INVEQ}.$$

SIMULATION RESULTS

Suppressing all identities written merely to aid presentation, this sector has four behavioral equations: (4.21), (4.25), (4.31), and (4.36). The identities (4.1) - (4.3) and (4.37) determine the stocks of the three goods and total fixed investment. Thus, these two variables do not appear in the presentation of our simulation results.

Treating the three stocks, the three investment categories, CDSTS, and INFXD as endogenous, a dynamic simulation of this sector was run over the period 1954 I - 1967 IV, the period for which the behavioral equations were estimated. The results are summarized in Table IV.2. They are not all that might be desired.

The RMS errors of the investment equations are all well above the estimated standard errors, and without exception go off the track and generate biased estimates. Clearly, in spite of all our attention to specification, the dynamic structure of the system has been poorly specified.

This is emphasized by the same experiment involving the four quarters of 1968. The results of which are summarized in Table IV.3. Predictions, as measured by absolute and relative RMS errors, are much improved. The bias problem exists, but it is not nearly as severe as in the earlier run. A comparison of the two experiments suggests that the investment sector tracks decently in short simulations, but that it will go off the track in long runs.

Table IV.2Investment Sector Simulation Results: 1954I - 1967IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
CDSTS	16.71	2.42	14.5	- 1.73	- 7.58 ***
INVH	22.62	2.60	11.5	- 1.87	- 7.68 ***
STKH	451.72	15.88	3.5	-13.55	-12.14 ***
INPL	18.52	1.28	6.9	- 0.83	- 6.32 ***
STKPL	224.11	5.74	2.6	- 5.12	-14.63 ***
INVEQ	33.59	3.75	11.2	3.21	12.88 ***
STKEQ	178.95	14.76	8.2	11.93	10.18 ***
INFXD	74.72	2.40	3.2	.52	1.65

*** Significant at 1%

Table IV.3Investment Sector Simulation Results: 1968I - 1968IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
CDSTS	17.89	1.71	9.6	-1.55	-3.72***
INVH	23.25	1.02	4.4	- .61	-1.29
STKH	534.97	.32	.06	- .18	-1.18
INPL	22.68	.93	4.1	- .73	- .14
STKPL	269.56	.54	.2	- .53	-8.87***
INVEQ	53.15	1.08	2.0	.31	.52
STKEQ	255.52	.27	.11	.09	.61
INFXD	99.13	1.81	1.8	-1.08	-1.29

*** Significant at 1%

CHAPTER IV - Fixed Investment Spending

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Chapter IV (cont'd)

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Appendix to Chapter IV

In the first section of this appendix, we shall discuss the estimated order-delivery lag for producer's durable equipment. In the second section, we derive the formulae employed to calculate the user cost of plant and of equipment, and we discuss the empirical counterparts of the theoretical constructs appearing in those relations.

I. The Delivery Lag in Producer's Durable Equipment

This analysis follows an article by Joel Popkin (1965). He noted that the machinery and equipment market classification of the Bureau of the Census corresponds closely to those industries which produce durable capital equipment. The producer's durable equipment expenditures series and the machinery and equipment industries' series overlap substantially. Differences in coverage should be noted. Producers' durable equipment includes investment in cars and trucks, which is not part of the machinery and equipment series. Unlike producers' durables, the shipment series includes exports but exclude imports of machinery and equipment. Nonetheless, the two series have moved quite similarly in the post-war period.

Seasonally-adjusted monthly data for new orders received and shipments made by the machinery and equipment industries is published by the

Bureau of the Census.* Both series were deflated by the wholesale price index for machinery and equipment industries, furnished by the Bureau of Labor statistics.**

As in the text, we assume a distributed lag relation between orders, ϕ , and shipments, S , is as follows:

$$(4A.1) \quad S = Q(L) \phi$$

where $Q(L)$ is a lag polynomial. Following Popkin and other writers, we assumed that $Q(L)$ is of the following form

$$(4A.2) \quad Q(L) = w_0 + w_1 L + \dots + w_T L^T,$$

where $\sum_{i=0}^T w_i = 1$, and $w_i = b_0 + b_1 i + \dots + b_k i^k$

That is, $Q(L)$ is treated as finite with length T , and with lag weights that can be approximated by a polynomial function of the lag. This is the scheme proposed by Shirley Almon (1965) and discussed briefly in Chapter II. It seems suitable for this problem and Almon (Ibid.) has used it successfully on similar data. A modification of this approach has been used by Tinsley (1967), Almon (1968), and Galper and Gramlich (1968).

It became apparent that T would need to be fairly large, so

* Data for 1953-1960 were taken from Table 3 of Current Industrial Reports, Series M3-1, Manufacturer's Shipments, Inventories and Orders (November 1964). Data for later years are from U.S. Bureau of the Census, Manufacturer's Shipments, Inventories, and Orders: 1961-1968 (Washington: U.S. Government Printing Office, 1968), Table 8.

** Results using the undeflated series were quite similar to those reported here.

estimates were fitted to the February, 1955 - December 1968 period.* We experimented with $k = 2$ and 4 , and $T = 4, 7, 12, 15, 18, 21$, and 24 . The polynomial coefficients, the b_i , were constrained so that $w_{T+1} = 0$. That is, we forced the lag weights to approach zero as i became large. Results obtained by further constraining $w_0 = 0$ were totally unsatisfactory. For both k 's, the best fitting equation assumed $T = 24$. The quadratic lag ($k=2$) exhibited a monotone decline in the w_i . In general $k = 2$ is a highly restrictive assumption which often distorts lag estimates. With $k=4$, however, the pattern was much more interesting. The lag weights were quite large initially and declined to essentially zero by the ninth month. There was then a second, although smaller, peak in the distributed later on, with the weights then declining to zero again. We thus obtained a bimodal lag distribution, with the two modes corresponding, we conjecture, to standard and specially made equipment. Almost all the estimates with $k=4$ showed this pattern.

The estimate with $k=4$ represented an improvement over the less general structure implied by $k=2$; the increase in the R^2 was significant at the 5% level. The w_i corresponding to this best estimate were as follows:

*

The reason for beginning with February will, hopefully, become clear below.

$w_0 = .1863$	$w_8 = .0139$	$w_{17} = .0256$
$w_1 = .1474$	$w_9 = .0094$	$w_{18} = .0284$
$w_2 = .1143$	$w_{10} = .0073$	$w_{19} = .0302$
$w_3 = .0886$	$w_{11} = .0071$	$w_{20} = .0306$
$w_4 = .0639$	$w_{12} = .0085$	$w_{21} = .0292$
$w_5 = .0457$	$w_{13} = .0112$	$w_{22} = .0258$
$w_6 = .0316$	$w_{14} = .0145$	$w_{23} = .0200$
$w_7 = .0211$	$w_{15} = .0183$	$w_{24} = .0115$
	$w_{16} = .0221$	

Even though there are 24 lagged months represented, the mean lag is only 6.86 months, with 54.6% of shipments occurring within four months of the receipt of orders.

The sum of the weights above is 1.0107. They were nomalized to sum to unity. We then obtained the quarterly weights shown in the text by assuming that the rate of orders and shipments were uniform during the quarter.

The procedure is best illuminated by an example in which a shorter lag distribution with the following monthly weights had been estimated.

$$\begin{array}{ll}
 w_0 = .4 & w_2 = .2 \\
 w_1 = .3 & w_1 = .1
 \end{array}$$

Supposing the rate of orders and deliveries is uniform within quarters, we reason as follows. Let S be current quarter's shipments, and let ϕ and $\phi(-1)$ be the orders placed in the current and previous quarters. The deliveries in the last month of the ^{current} quarter come from orders placed in the current quarter and the last month of the previous quarter. We can write this as

$$S/3 = .4\phi/3 + .3\phi/3 + .2\phi/3 + .1\phi(t-1)/3$$

Similarly, we have the equations for the first and second months of the quarter:

$$S/3 = .4\phi/3 + .3\phi/3 + .2\phi(-1)/3 + .1\phi(t-1)/3$$

$$S/3 = .4\phi/3 + .3\phi(t-1)/3 + .2\phi(t-1)/3 + .1\phi(t-1)/3$$

Adding these three equations,

$$S = (2/3)\phi + (1/3)\phi(t-1),$$

so the quarterly lag weights are $(2/3)$ and $(1/3)$.

Notice that with four monthly weights, or $1 + 3N$ in general, this process comes out "even"; all orders placed in all months of the earliest quarter considered are accounted for. The twenty-five monthly lag weights were converted into the following quarterly lag weights using this approach:

$w_0 = .319$	$w_3 = .032$	$w_6 = .082$
$w_1 = .266$	$w_4 = .028$	$w_7 = .083$
$w_2 = .102$	$w_5 = .054$	$w_8 = .033$

Notice that the second mode of the monthly distribution is reflected in a second mode here. The mean lag of this structure is 2.29 quarters, almost exactly the mean lag of the monthly distribution.

II. The User Cost of Capital

Our treatment of the user cost of capital basically follows the early work of Jorgenson (1963, 1965).^{*} There is some difference in detail, however. First, the underlying theory will be portrayed for a world without taxes. Second, this will be modified to account for corporate income taxes. Third, the operational procedures used for measuring user cost for plant and for equipment are presented.

Basic Theory

Using Jorgenson's notation, we define the following quantities for a "representative" firm:

- L = labor hired
- K = effective capital stock employed
- I = gross investment
- Q = output = $Q(K, L)$
- \bar{s} = wage rate at time zero
- \bar{q} = price of capital goods at time zero
- \bar{p} = price of output at time zero
- r = discount rate (cost of capital funds)
- μ = expected rate of increase of all prices
- δ = rate of deterioration of the capital stock

*

In his later work, Jorgenson uses the concept of quasi-rents instead of user cost. See Hall and Jorgenson (1967) and Bischoff (1968).

Observe that we have assumed that the firm expects no change in relative prices. The following identity relates investment to changes in the effective capital stock:

$$(4A.3) \quad \dot{K} = I + \delta K,$$

where $\dot{K} = dK/dt$. Thus the larger is δ , the larger gross investment must be if the effective capital stock is to be kept constant.

The firm's problem is to choose gross investment and labor inputs at all points in time so as to maximize the present discounted value of its net cash inflows. This may be expressed algebraically as

$$(4A.4) \quad \max_{I, L} \int_0^{\infty} (\bar{P}Q - \bar{s}L - \bar{q}I) e^{-(r-\mu)t} dt = \int_0^{\infty} J(t) dt.$$

Substituting for I from (4A.3), the Euler necessary conditions for a maximum of the integral above may be applied. (See Allen (1962), Chapter 20, and Gelfand and Fomin (1963), Chapter 1.)

These yield the following equations, which must hold at all points in time:

$$(4A.5) \quad \partial J / \partial L = 0; \quad \partial Q / \partial L = \bar{s} / \bar{P}.$$

$$(4A.6) \quad \partial J / \partial K - d/dt (\partial J / \partial K) = 0; \quad \partial Q / \partial K = (q/P)(\delta + r - \mu).$$

Equation (4A.5) is just the familiar condition that the marginal product of labor equal the wage rate. Equation (4A.6) is also a marginal productivity condition, but instead of the purchase price of capital goods, the relevant quantity is the implicit rental value or user cost. The term on the far right of (4A.6) is the money rate of interest, (r) , minus the own-rate of interest of capital goods, $(\mu - \delta)$. This difference is the required rate of return for new gross investment.

We will not work with all terms which appear in the expression for user cost, but only with the required rate of return. There are three reasons for ignoring the relative price term. The first is that published deflators for plant and equipment are thought by many to be badly biased and crude at best. The second reason is that these price indexes do not properly account for technical change. Finally, the value of the model would be seriously compromised were it forced to depend on (almost inevitably) poorly specified relative price equations.

In a world without taxes, one would want required rates of return for each capital good which would differ only in the appropriate rate of deterioration employed, and to each would correspond an equation like (4A.6).

Taxes

Modifications must be made in this model to allow for taxes.

Again, following Jorgenson, we define the following tax parameters:*

u = corporate tax rate

v = fraction of actual deterioration of capital stock that can be deducted for tax purposes

w = fraction of (rqK) that can be deducted for tax purposes

x = fraction of capital gains (on capital) that can be deducted for tax purposes.

c = rate of investment credit for tax purposes.

Jorgenson assumes that the firm is trying to maximize the net present value of its cash inflows. Questions of debt-equity mix and stockholder welfare are ignored. This is indeed the simplest course to follow, and we shall also assume that the capital structure is given to us by prior considerations. By assuming that the firm is a price-taker, Jorgenson assumes no shifting of the corporate income tax, a position we shall adopt here too. **

*

This model does not recognize the case where investment tax credits must be deducted from the depreciation base, a situation which held only from 1962 III to 1963 IV and is not of great importance.

** See Gordon (1967) for evidence on this point.

Formally, the problem facing our representative firm may be stated as follows:

$$(4A.7) \quad \max_{I, L} \int_0^{\infty} J'(t) dt, \text{ where}$$

$$J' = J - u[\bar{P}Q - \bar{s}L - \bar{q}K(v\delta + wr - x\mu) - c\bar{q}I]e^{-(r-\mu)t}$$

The two Euler conditions now yield the following necessary conditions for a maximum of this integral:

$$(4A.8) \quad \partial Q / \partial L = (s/P)(1-u).$$

$$(4A.9) \quad \partial Q / \partial K = (q/P) \left[\frac{1-c-uv}{1-u} \delta + \frac{1-c-uw}{1-u} r - \frac{1-c-ux}{1-u} \mu \right]$$

These are identical to the conditions derived by Jorgenson in the papers cited, except for the explicit treatment of the investment tax credit. The expression in brackets in equation (4A.9) is the correct required rate of return. Since capital gains or losses cannot be deducted, we have $x=0$ immediately, and we can write the required rate of return as

$$(4A.10) \quad R = \left[\frac{1-c-uv}{1-u} \right] \delta + \left[\frac{1-c-uw}{1-u} \right] r - \left[\frac{1-c}{1-u} \right] \mu$$

$$= WD\delta + WRr - WI\mu.$$

We now consider how best to attach values to the variables appearing in this equation. Each element in Equation (4A.10) will be examined in turn, starting with those that would be present in a world without taxes. Then the tax parameters which will cause the values of the W 's to depart from unity will be evaluated.

Deterioration Rates

Hall and Jorgenson (1967) estimate the rate of deterioration, δ , as 2.5 times the reciprocal of the Bulletin F lifetime.* Denoting this lifetime as L, this is equivalent to assuming that a fraction f of original effective capital remains at the end of L years, where f of original effective capital remains at the end of L years, where f is given by

$$(4A.11) \quad f = (1 - \frac{2.5}{L})^L.$$

For the categories of capital they consider, f falls between 6.2 and 7.4 per cent. These percentages seem low, and they vary with the lifetime considered.

We calculated L's from the Hall-Jorgenson deterioration rates. We then averaged the figures for manufacturing and non-manufacturing plant and equipment, using the net stock in each sector in 1960 as weights.**

The resultant average lifetimes were 38.1 years for plant and 14.3 years for equipment. Assuming that 10% of original capacity remains after these periods, the rates of deterioration emerge from (4A.12):

$$(4A.12) \quad .10 = (1 - \delta)^L.$$

These calculations yielded annual rates of deterioration of 5.86% for plant and 14.8% for equipment, as shown in the text.

* This Bulletin was originally issued by the U.S. Treasury as a guideline to businessmen on plant and equipment lifetimes that were acceptable for tax depreciation.

** See Lawrence Grose, Irving Rottenberg, and Robert C. Wasson, "New Estimates of Fixed Business Capital in the United States, 1925-65," Survey of Current Business, 46(December, 1966): 34-40, and additional tables in Survey of Current Business, 47(February, 1967) pps.20-24.

Cost of Capital

Most previous work has taken as the cost of capital, r , some measure of the rate of interest on corporate debt. Corporations raise most of their funds from internal sources or new stock issues, however, and these sources of funds must be valued at the cost of equity. In measuring this cost, we follow Gordon and Shapiro (1956). If the dividend yield on a stock is D/P , and dividends are expected to grow at a rate g^* , the market is discounting future dividends at a rate given by

$$(4A.13) \quad r = \frac{D}{P} + g^*.$$

A discrete analog of this formula was used to determine the cost of equity. The dividend yield used was the published quarterly series for Standard & Poor's 500 common stocks taken from various numbers of the Economic Report of the President. Dividend payments in each quarter were taken from the Survey of Current Business; the series used appears as BDIV in Chapter VII. The quarter-to-quarter rate of growth of dividends was geometrically smoothed to obtain an estimate of the expected growth rate of dividends in each quarter. It was assumed that actual dividend growth from 1929 to 1930 equaled expected dividend growth, and the annual rates of growth were smoothed until 1946 I, when quarterly figures became available. From that point on, the following equation was employed, where $g(t)$ is the actual rate of growth in period t , $g^*(t)$ is the expected rate, and a is the quarterly smoothing constant.*

* The equation used for annual rates of growth before 1946 is the same as (4A.14), except that (a) must be replaced by $(1-(1-a)^4)$.

$$(4A.14) \quad g^*(t) = (1-a) g(t) + ag^*(t-1)$$

We then substituted the $g^*(t)$ thus obtained and the actual dividend yield into (4A.13) to obtain a cost of equity for each quarter. We then averaged this series over the period 1948-1966. Smoothing constants of .3, .5, .7, and .9 were employed in these computations. The computed average cost of equity was approximately the same for all smoothing constants. Since we expect a priori that expectations about long-run rates of growth of dividends are slow to change, we used $a = .9$ in the final computations. This figure yielded an average after-tax cost of equity of 12.09% per annum from 1948 through 1966. This is 3.28 times the cost of debt, which we take as the yield on AAA corporate bonds, RCB.

It is a well-known result that if funds are raised in fixed proportions from two or more sources, as its discount rate, the corporation should use a weighted average of their costs, the weights being the fractions of total funds raised from each source.*

From 1954 through 1966, U.S. nonform nonfinancial corporations raised 32% of funds from debt and 68% from internal sources or new stock issues.** The appropriate discount rate was thus calculated as the

* See E. Solomon (1963).

** See The Economic Report of the President; 1966, p.287; 1967,p294.

weighted average of the cost of debt and the cost of equity, which we take as 3.28 times the cost of debt, using these percentages as weights. Formally,

$$(4A.15) \quad r = .68 (3.28 \text{ RCB}) + .32 \text{ RCB} = 2.55 \text{ RCB},$$

where RCB is the cost of debt. The cost of capital, r , was thus computed as 2.55 times the yield on AAA corporate bonds.

Expected Inflation

To measure the rate of inflation expected by businesses, we looked at the quarter-to-quarter changes in the implicit deflator for private product, PGPP, expressed at annual rates. We used exponential smoothing, as described above, with a quarterly smoothing constant of .9.^{*} The expected rate of inflation, μ , is thus estimated by a weighted average of current and past percentage changes in the deflator for gross private product, the weights declining as $(.9)^T$. The formula for this estimate, IN90, is

$$(4A.16) \quad \text{IN90} = 40. \left[(\text{PGPP} - \text{PGPP}(t-1)) / \text{PGPP}(t-1) \right] + .90 \text{ IN90}(t-1).$$

Corporate Tax Rate

The rate of corporate taxation, u , was taken as the ratio of corporate profits taxes to corporate profits, using Survey of Current Business figures for both. This variable appears explicitly in the model as BCPTRT; see Chapter VII for more details.

Depreciation Deductions

We assumed that the change in the depreciation laws in 1954 was not designed primarily to encourage investment, but rather to make the rate of depreciation allowable for tax purposes equal to the rate of deterioration.

* This implies long lags in the formation of price expectations. See Feldstein and Eckstein (1970) and the literature there cited for evidence on this point.

We thus take v equal to unity from the first quarter of 1954 through the second quarter of 1962.

The calculation of v for other periods, was based on the ratio of the corporate capital consumption allowance, deflated by the implicit deflator in the Survey of Current Business for fixed, non-residential investment, to the net capital stock, taken from the Survey of Current Business as described above. For the period 1948-52, this ratio was .0629, while in 1956-59 and 1963-66, it was .0755 and .0977. Hence v is .83 before 1954 and 1.29 after accelerated depreciation was enacted in 1962 II.

Interest Deductions

If interest payments are taken to be $(.32)RCB_{qk}$, from equation (4A.15), we find that w , the ratio of interest payments (which are deductible) to r_{qk} is .125. If changing prices were explicitly recognized, the value of w would be slightly smaller, since some borrowing in the past was undertaken to finance plant and equipment when they were cheaper, and the amount of debt outstanding will be less than $(.32)qK$. But since w is already small, this would be a very slight adjustment, so it was ignored.

Investment Tax Credit

Up to this point, the discussion has applied equally to plant and equipment, with the exception of the different rates of deterioration. The investment tax credit, in effect from 1963 III - 1966 III and in 1968, applied only to purchases of equipment. Notice that the variable c enters all the weights in (4A.10) so we must distinguish between the weights for plant and those for equipment in the periods when the credit was in force.

We follow Bischoff (1968) and let $c = .05$ for the equipment weights WDE , WRE , and WIE in the periods when the credit was in force. This variable is always zero in the computation of WDP , WRP , and WIP .

APPENDIX TO CHAPTER IV

Estimating the Order-Delivery Lag for Producer's Durable Equipment

Almon, S. "The Distributed Lag Between Capital Appropriations and Expenditures," Econometrica, 33(January, 1965): 178-196.

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Appendix to IV (cont'd)

Shifting of the Corporation Income Tax

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Inflationary Expectations

Feldstein, M. and Eckstein, O. "The Fundamental Determinants of the Interest Rate," Review of Economics and Statistics, 52(November, 1970): 363-375.

CHAPTER V

Change in Business Inventories

This sector is uncomplicated since only one endogenous variable, inventory investment is determined here. Yet it is by no means easy to explain statistically changes in inventories. The main reasons for this are that available data do not permit the kind of disaggregation that seems essential for structural modeling, and problems of valuation and price deflation that are especially severe for inventories.

The data series used in this sector are listed in Table V.1. All were taken from the Survey of Current Business. INBIN and YGPP are measured in billions of constant dollars, seasonally-adjusted at annual rates.

Inventories are of three kinds: finished good stocks, stocks of raw materials, and goods in process. Most formal theories of inventory holding relate only to final goods inventories, while there is reason to suspect that the other categories have basically different dynamics. Unfortunately, INBIN is not broken down by type of inventory, so the three categories must be pooled together.* This is one major reason why most models of inventory behavior, ours included, have such low predictive power. Since short-run inventory fluctuations are a major source of cyclical instability, this lacunae is a source of major weakness in the formation and execution of countercyclical government actions.

* Disaggregation by type of inventory is possible at the manufacturing level, but not for total inventory investment which appears in the National Income Accounts.

Table V.1Variables Used in Inventory Sector

(Endogenous - Determined in Inventory Sector)

INBIN Change in Business Inventories.

(Endogenous - Determined Elsewhere in the Model)

YGPP Gross Private Product

All National Income Figures are seasonally adjusted at Annual Rates

A variety of inventory models use total unfilled orders as an explanatory variable, but ours does not. First, if unfilled orders appear here, they must be explained elsewhere in the model. And this is by no means an easy task, since a great deal of new orders are truly exogenous. Further, if a manufacturer accumulates unfilled orders for a particular good, he is clearly not holding a finished goods inventory stock. The level of unfilled orders thus can only influence directly raw materials and goods in process inventories (though it may affect the finished goods stocks in other lines via expectations). Putting this variable in an equation for total inventories seems, to a certain extent, to be sweeping a real aggregation problem under the rug.*

Attempts were made to use the Galper-Gramlich (1968) estimates of delivery lags to construct a series of estimated inventory changes originating in defense production. This variable was never significant at the 10% significance level. Finally, a dummy variable designed to account for the effects of steel strikes on inventory accumulation also failed.**

Inventory equations that appear in most econometric models are based on Lovell's⁽¹⁹⁶¹⁾ adaptation of the standard stock adjustment models: investment is assumed to adjust to the difference between desired stocks and actual stocks at the period's outset. Desired stocks in turn are assumed proportional to sales or production.

* David Belsley's excellent study, Industry Production Behavior: the Order-Stock Distinction, (North-Holland Publishing Company, Amsterdam, 1969) using monthly two digit manufacturing inventory data shows what can be done about inventory investment at that level.

** The unfilled orders and Galper-Gramlich lag procedure experiments were carried out in connection with the stock adjustment model, not the distributed lag model finally adopted.

In symbols, let

I_t = Inventory investment

K_t = Inventory Stock at end of period

X_t = Sales (or output)

β = Desired inventory sales ratio

K_t^* = Desired Inventory Stock at end of period

α = Reaction coefficient

The basic equilibrium relation is between K_t^* and S_t :

$$(5.1) \quad K_t^* = \beta_t$$

The dynamic adjustment process is simply the following:

$$(5.2) \quad I_t = \alpha (K_t^* - K_{t-1}).$$

When (5.1) is substituted into (5.2), we have the standard stock adjustment equation:

$$(5.3) \quad I_t = \alpha \beta X_t - \alpha K_{t-1}.$$

Its explanatory variables are inventory stock at the beginning of the period, private gross national product in constant dollars and perhaps, the change in output to reflect expectations or involuntary inventory depletion or increase.

After exploring this formulation, we rejected it in favor of straightforward distributed lag acceleration model. The stock adjustment

model described above is one of several ways of expressing dynamic adjustments of a business stock to output fluctuations - the more elaborate ones are in the plant, equipment and housing equations in Chapter 4. Stock adjustment models are the simplest while simple distributed lags of the form:

$$(5.) \text{ INBIN} = a_0 \Delta \text{YGPP}_t + a_1 \Delta \text{YGPP}(t-1) + a_2 \Delta \text{YGPP}(t-2) + \dots + \epsilon_t$$

are intermediate in complexity. This form was selected because of better fit, lower serial correlation and a priori reasonable coefficients. The speed of adjustment in the estimated stock adjustment model was .06, which seems excessively low. Since the sum of the coefficients in (5.1) equals the desired capital-output ratio or capital coefficient, still another comparison is possible, since the actual average capital-output ratio should approximate the planned capital coefficient.

The final equation was estimated using Almon's (1965) technique for estimating distributed lags.* The estimated lag weights are:

lag weights		t statistic
a_0	= .3352	5.13
a_1	= .2107	6.08
a_2	= .1512	3.94
a_3	= .1334	5.38
a_4	= .1339	3.56
a_5	= .1294	3.83
a_6	= .0964	1.53

* See Chapter II for a discussion.

The coefficients follow a logical, declining pattern.* The mean lag is 2.2 quarters which seems highly reasonable. Over two-thirds of the total effect appears within the first year. The coefficients sum to 1.19, but since all the quarterly data were multiplied by four to put them on annual basis, the correct estimate of the capital coefficient, β , is $1.19/4 = .30$. Although the actual capital coefficient has, apart from cyclical variations, remained close to .20, the order of magnitude is close enough to provide minimal reassurance.**

*The Almon equation from which the lag weights were obtained was a third degree polynomial with $R^2 = .728$, $SE = 2.44$ and $DW = 1.41$. This compares to the stock adjustment equivalent which had $R^2 = .624$, $SE = 2.97$ and $DW = .908$. A straightforward regression of INBIN on lagged YGPP otherwise similar (no intercept, seven quarter lags) had highly similar test statistics, but the lag pattern was considerably more erratic. Although the practical differences are negligible, a desire for good theoretical properties favors the Almon form. Shorter periods (four or five quarters, not shorter) possess similar test statistics, but the lag weights were U-shaped (large, small, then large again) which is somewhat implausible.

** Using current dollar business inventories (Source: Business Statistics, 1967, U.S. Department of Commerce, page 22, Total Manufacturing and Trade) and current dollar gross private product, the ratios were .215, .221, .191 and .198 in 1954, 1957, 1964 and 1968. Those inventory figures are not consistent with INBIN.

The selected equation explains 73% of the variance in inventory accumulation, and the Durbin-Watson statistic suggests some positive serial correlation of the errors. Few aggregate inventory equations noticeably outperform it, though. It is difficult to imagine that much progress will be made in this area until INBIN can be separated into its logical components. Even then, arbitrary accounting elements in the valuation of inventories plus the large errors arising from price correcting the current dollar stock figures will place a severe limit on attainable accuracy. Remember that inventory investment is a small difference between the stocks of two adjacent periods.

Simulation results were calculated only since there are no lagged endogenous variables for 1968. As shown in Table V.1, the bias is small but the error variance is large. These results are broadly consistent with the estimated parameters. Inventories have large, nearly random quarter to quarter changes which cannot be readily characterized. This sector is likely to remain a major unpredictable element in macroeconomic models, since random shocks in demand are heavily transmitted into inventory investment via buffer stock motivation.

TABLE V.1

Change in Business Inventories Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	1968 I	1968 IV	<u>t-test for Bias</u>
			<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	
INBIN	6.67	3.04	45.6	1.32	.83

CHAPTER V - Change in Business Inventories

An Elegant Theoretical Treatment of the Role of Inventories

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A Survey of Aggregate Inventory Equations

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The Stock-Adjustment Approach

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Defense Production and Goods-in-Process Inventories

Galper, H., and Gramlich, E. "A Technique for Forecasting Defense Expenditures," Review of Economics and Statistics, 50 (May, 1968): 143-155.

The Distributed Lag Formulation Employed

Almon, S. "The Distributed Lag Between Capital Appropriations and Expenditures," Econometrica, 33(January, 1965): 178-196.

CHAPTER VI

International Trade

International trade does not have much effect on the level of aggregate economic activity in the U. S. economy. Hence our model has been endowed with only the most rudimentary trade sector.

DATA

Table VI.1 exhibits the variables employed in this sector. All quantities except DDSTR, a dummy variable measuring the impact of dock strikes, were taken from the Survey of Current Business. The flow variables, TIM, TRBAL, TEX, and YGPP, are all expressed in billions of constant dollars, seasonally-adjusted at annual rates.

Information on the length and location of dock strikes was taken from various Bureau of Labor Statistics Bulletins beginning with No.1184 and terminating with No.1339, and from the U. S. Transportation Task Force, Longshore Strikes (Washington: Government Printing Office, 1970). The country is divided into four regions: New York - New Jersey, Other East Coast, Gulf Coast, and West Coast. The fraction of total trade tonnage moved through each of these was computed for the years 1957 and 1964, when there were no strikes. As the fractions from the two years were nearly identical, they were averaged for the purpose of constructing DDSTR. The weights thus obtained for the four regions are .15, .43, .31, and .11, respectively. The dummy for each quarter was constructed by computing the fraction of the quarter that each region's ports were closed by strikes, multiplying by the weight for (importance of) that region, and adding across regions. The

Table VI.1

Variables Used in Trade Sector

(Endogenous - Determined in Trade Sector)

TIM	Imports
TRBAL	Trade Balance.

(Endogenous - Determined Elsewhere in the Model)

YGPP	Gross Private Product.
PGPP	Implicit Price Deflator for Gross Private Product.

(Exogenous Variables)

TEX	Exports
PTIM	Implicit Price Deflator for Imports.
DDSTR	Dock Strike Dummy Variable.

dummy thus represents the fraction of "normal" imports that might be held out by dock strikes. In the sample period, DDSTR is greater than .20 only in the first quarter of 1963 and the first quarter of 1965. The dummy's effect should be multiplicative, since it is logically multiplied by "normal" imports to obtain a measure of dollar impact.

EQUATIONS The first equation in the trade sector is the identity connecting exports, imports, and the trade balance:

$$(6.1) \quad \text{TRBAL} = \text{TEX} - \text{TIM}$$

The sector needs only an equation for TIM, net imports to be complete since exports are exogenous.

Clearly imports should depend on DDSTR. If the dependence is multiplicative, the log-log form is suggested. The level of economic activity should also influence imports. Both real gross private product and real disposable personal income were tried as activity variables. Finally, the ratio of PTIM to domestic prices should affect imports. Both the implicit deflator for personal consumption and PGPP were tried as measures of the domestic price level. All equations estimated allowed for distributed lags of the Koyck form. The best equation for TIM was

Import Demand

$$(6.2) \quad \log(\text{TIM}) = -1.871 + .4831 \log(\text{YGPP}) - .2192 \log(\text{PTIM/PGPP}) \\ (4.28) \quad (4.56) \quad (1.78) \\ + .6590 \log[\text{TIM}(t-1)] + .1352 \log(1. - \text{DDSTR}) \\ (8.48) \quad (2.98)$$

$$\begin{aligned} R^2 &= .990 \\ SE &= .0268 \\ SE(\text{Antilog}) &= 1.072 \\ DW &= 1.753 \end{aligned}$$

Even though the relative price term was not significant at the 5% level of significance, we retained it in the equation since it is significant at 10%, has the right sign, and theory strongly suggests it belongs there.

The short-run income elasticity of imports is .48, while the long-run elasticity is 1.42. Corresponding price elasticities are -.22 and -.64. The long-run income elasticity agrees almost exactly with the more complete estimate of Houthakker and Magee (1969), but their estimate of the long-run price elasticity is -.88.* The difference should not be cause for concern, since different data series were used. Our short-run price elasticity corresponds closely to that of Houthakker and Magee, which is -.20, although their short-run income elasticity of .38 exceeds ours noticeably. The mean lag of this equation is 1.93 periods.

The term involving the dock strike dummy requires further explanation. Let TIM^* be the volume of imports that would take place without any dock strikes. Taking the antilogs of both sides of (6.2) will then yield

$$(6.3) \quad TIM = TIM^* (1.-DDSTR)^{.1352}.$$

Clearly if DDSTR were one, there would be no imports, while if DDSTR = 0, TIM equals TIM^* . As it should, (1-DDSTR) has an exponent less than one. If only part of the country is struck, the volume of imports entering through other ports will increase to compensate. Similarly, if a dock strike ties

* Note that sampling errors in the long-run estimate are magnifications of the short-run errors, since the long-run estimate is obtained as the ratio of two random variables (estimated coefficients) for which the denominator is often quite small. Thus the standard error of the short-run price elasticity is .1235 while that of the long-run price elasticity is more than three times larger, .393.

up a given port for only part of a quarter, the port will be busier than normal at other times during the quarter. Thus if DDSTR is equal to, say, .10, we would expect more than 90% of TIM^* to be imported. And this is just what the exponent of less than one implies. Only for large values of DDSTR will imports be noticeably disturbed.

A model consisting of equations (6.1) and (6.2) was simulated over the sample period, 1954 I - 1967 IV, and over the four quarters of 1968. The results are shown in Table VI.2.

The results for the sample period seem quite acceptable. Large relative errors are made in the trade balance, since exports and imports are nearly equal. There is no noticeable bias in this period. This sector performs quite badly in 1968, significantly underestimating imports.

Table VI.2

Trade Sector Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
(1954 I - 1967 IV)					
TIM	24.9	.95	3.96	.06	.66
TRBAL	5.44	.95	17.5	- .06	-.66
(1968 I - 1968 IV)					
TIM	44.73	3.44	7.7	-3.33	-6.63***
TRBAL	.93	3.44	357.	3.33	6.63***

*** Significant at 1%.

The Demand for Imports

- Ball, R. J., and Marwah, K. "The United States Demand for Imports, 1948-1958," Review of Economics and Statistics, 44 (November, 1962): 395-401.
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CHAPTER VII

Income Distribution

This chapter describes the most elaborate sector of the model, consisting of 13 behavioral equations and 7 identities. The "job" of the income sector is to determine disposable personal income.

A legitimate question is why so many equations are needed to determine this one quantity, since many models use far fewer equations to do the job. But we feel that in so doing these models neglect strategic exogenous variables that influence the difference between gross national product and disposable income and which are essential to a model designed for structural and policy relevance. In particular, important policy parameters such as tax and transfer rates are often neglected to obtain a more condensed description of the distribution of income.

We thus felt obliged to undertake detailed modeling of the income side of the income and products accounts. This approach forces us to live with a number of less than satisfactory equations (rather than leave them implicit) but it does serve to illustrate the structure of the relations determining disposable income.

Many of the equations in this chapter have weak theoretical bases. The reasoning behind each equation will be stated, but it must be remembered that the detailed tax and accounting rules behind many of the aggregate quantities are much more complex.

THE DATA

The data appearing in this sector are listed in Table VIII.1. Unless indicated otherwise, all series are measured in billions of current dollars, seasonally-adjusted at annual rates. All of the endogenous variables determined in this sector were taken directly from the Survey of Current Business, except for YNIP and YPRCE. These two quantities were computed from data in the Survey, as indicated. Their definitions as given in the Table will be clearer if it is recalled that Gross Government Product, YGGPI, is equal to compensation of government employees.

The first three endogenous variables determined in other sectors were also taken from the Survey. The quantity LUT is a quarterly average of seasonally-adjusted monthly figures taken from Employment and Earnings. The two interest rates are quarterly averages of monthly figures taken from the Federal Reserve Bulletin. STBIN is equal to the sum of all past business inventory investment, starting in 1947I. This variable is discussed at length in Chapter V. PEHRS is an unpublished series, estimated and furnished to us by the Bureau of Labor Statistics; it is seasonally adjusted. The private wage rate, WRPVT, was computed according to the following identity:

$$(7.1) \quad \text{WRPVT} = 1000 \cdot \text{YPRCE} / \text{PEHRS}.$$

The constant corrects for the difference in units.

The first three exogenous variables are from the Survey of Current Business, and the fourth, TIME, is obvious. Total population was taken

TABLE VII .I

Variables Appearing in Income Sector

(Endogenous - Determined in Income Sector)

YNIP	Private National Income (defined as national income minus gross government product).
BCCA	Capital Consumption Allowances.
BIBT	Indirect Business Tax and Non-Tax Liability.
BTRF	Business Transfer Payments.
BPBT	Corporate Profits and Inventory Valuation Adjustment.
YPRCE	Private Compensation of Employees (defined as compensation of employees minus gross government product).
YPTOT	Proprietors' Income.
YRENT	Rental Income of Persons.
NETINT	Net Interest.
BCP	Corporate Profits (excluding inventory valuation adjustment).
BCIVA	Inventory Valuation Adjustment.
BCPT	Corporate Profits Tax Liability.
YPERS	Personal Income.
BDIV	Corporate Dividends.
GEINT	Interest Paid by Government.
GETRFP	Government Transfer Payments to Persons.
GRFICA	Contributions for Social Insurance.
YCING	Interest Paid by Consumers.
YDPI	Disposable Personal Income.
GRPTX	Personal Tax and Non-Tax Payments.

(Table VII .I, Continued)

(Endogenous - Determined Elsewhere in the Model)

YGPPI	Gross Private Product.
PGPP	Implicit Deflator for Gross Private Product (1957-59 = 100
YGGPI	Gross Government Product.
LUT	Total Number Unemployed (millions of persons).
RCB	Average Yield of Moody's AAA Corporate Bonds (percent).
RCP	Average Yield on 4-6 Month Prime Commercial Paper (percent).
STBIN	Four Times the Stock of Business Inventories on Hand at the Start of the Quarter, Minus the Stock on Hand at the Start of 1947I (billions of constant 1958 dollars).
WRPVT	Private Sector Average Wage Rate (dollars per hour).
PEHRS	Total Man-Hours Worked in the Private Sector (millions of hours, annual rates).

(Exogenous Variables)

GRSUB	Subsidies less Current Surplus of Government Enterprises.
STADIS	Statistical Discrepancy.
WSACCR	Wage Accruals Less Disbursements.
TIME	Time Trend (equals 1.0 in 1954I, rises by 1.0 each quarter).
LTPOP	Total Non-Institutional Population (millions of persons).
BCPTRT	Observed Corporate Profits Tax Rate (fraction).
LRPOP	Civilian Population Aged 65 and Over, 62 and Over for Females Since 1956 (millions of persons).
CGBTRT	First Bracket Federal Tax Rate of Wages and Salaries (fraction).
GDBT	Marketable Federal Debt of all Maturities Outside Federal Reserve and U.S. Government Agencies and Trust Funds (billions of dollars).
TVMOA	Maximum Per-Family OASI Benefit per Month (dollars).
TXRSS	Employer and Employee Tax Rate for OASI (percent).
TXRUB	Employer's Tax Rate for Unemployment Insurance (percent).
TMXEPE	Index of Maximum OASI-Taxable Earnings Per Employee (1958 = 100).

All National Income Figures are Seasonally Adjusted at Annual Rates

as the average of monthly figures from the Survey of Current Business. BCPTRT, treated as exogenous, is the ratio of BCPT to BCP. Figures for the population eligible for Social Security, LRPOP, were furnished by the Brookings Institution, as were all the remaining series. GDBT is a seasonally-adjusted quarterly average. Our debt to the Brookings Institution is considerable.

THE EQUATIONS

a. Private National Income

Equations in this sector will be described roughly in the order that the corresponding variables appear in the usual tracing-out of the relationship between gross national product and disposable income. We begin with the identity that determines private national income:

$$(7.2) \quad \text{YNIP} = \text{YGPPI} - \text{BCCA} - \text{BIBT} - \text{BTRF} + \text{GRSUB} - \text{STADIS}.$$

Of the quantities on the right, only BCCA, BIBT, and BTRF are determined in this sector.

Capital Consumption

Capital consumption allowances depend on the original-cost value of the capital stock, on its age composition, and on the prevailing depreciation laws. The major change in these laws in the sample period occurred in 1962, but a variety of attempts to capture the effects of this change through dummy variables failed. Our final equation for this quantity is

$$(7.3) \quad \text{BCCA} = \underset{(2.66)}{.0073} \text{ YGPPI} + \underset{(32.2)}{.9392} \text{ BCCA (t-1)}$$

$$\begin{aligned} R^2 &= .999 \\ SE &= .404 \\ DW &= 1.67 \end{aligned}$$

The lagged dependent variable reflects the influence of previously installed capital, while the income term is a proxy for the amount of newly installed capital. We should have used business fixed investment in current dollars, but this would have required an equation for the plant and equipment implicit deflator exclusively for this limited purpose.

Indirect Business Taxes

Indirect business taxes consist of sales and property taxes. It seemed sensible to relate the change in BIBT to both the change in gross private product and its level. The first term is designed to pick up the effects on sales taxes of the change in sales, while the second is related to the change in the stock of tangible assets. The remark made just above about fixed investment holds here also. The equation used is

$$(7.4) \quad [\text{BIBT} - \text{BIBT(t-1)}] = \underset{(2.73)}{.0328} [\text{YGPPI} - \text{YGPPI(t-1)}] + \underset{(5.11)}{.0011} \text{ YGPPI}$$

$$\begin{aligned} R^2 &= .296 \\ SE &= .441 \\ DW &= 1.89 \end{aligned}$$

The equation fit is quite acceptable: the R^2 is small only because the dependent variable is a first difference. Were we to calculate R^2 for

levels the R^2 would exceed .90.* The standard error of \$441 million is rather small relative to the mean value of BIBT, which was \$47.81 billion in the sample period.

Business Transfer Payments

Finally, business transfer payments average only about two billion dollars, and they appear related to the level of economic activity. The following simple equation is used to explain this variable:

$$(7.5) \quad BTRF = -.2610 + .0019 YGPPI + .6750 BTRF (t-1) \\ (3.82) \quad (3.97) \quad (7.69)$$

$$\begin{aligned} R^2 &= .994 \\ SE &= .048 \\ DW &= 2.31 \end{aligned}$$

b. Corporate Profits, Dividends, and Other Non-Wage Income

Given private national income, we need to determine corporate profits and dividend payments in order to get personal income. Corporate profits (including inventory valuation adjustment) are calculated as a residual, by subtracting all the other components of private national income from the total. The identity used is the following;

$$(7.6) \quad BPBT = YNIP - YPRCE - YPTOT - YRENT - NETINT$$

Behavioral equations for the last four quantities on the right of this equation must be estimated.

* This first difference equation has no intercept. Since no linear trend term appears in the original equation, consistency in specification calls for suppressing the intercept here. The original intercept, of course, vanishes upon first differencing.

Private Compensation of Employees

Private compensation of employees is given by a re-arrangement of identity (7.1), the equation used to compute WRPVT:

$$(7.7) \quad \text{YPRCE} = (\text{WRPVT PEHRS})/1000.$$

The private wage rate is determined in the wage-price sector (Chapter X), and PEHRS is determined in the labor sector (Chapter IX).

Proprietors' Income

Proprietors' income is a declining share of gross private product. We explain it with an equation that involves gradual motion towards an exponentially decaying share:

$$(7.8) \quad \log(YPTOT/YGPPI) = -.5200 + .7550 \log[YPTOT(t-1)/YGPPI(t-1)] - .0014TIME$$

(2.72)
(8.34)

	R^2	=	.974
	SE	=	.0161
Antilog	SE	=	1.0162
	DW	=	1.66

This equation fits well enough to permit its use in deriving a usable corporate profit total. The trend coefficient implies that proprietors' share is declining by about one-half of one per cent every year.

Rental Income

The share of gross private product accounted for by rental income has also been declining. The equation for YRENT in the model is

Rental Income

$$(7.9) \quad (YRENT/YGPP) = \frac{.0007}{(1.52)} + \frac{.9790}{(73.42)} (YRENT/YGPP)_{(t-1)} + \frac{.0861}{(9.29)} (LTPOP/YGPP)$$

$$\begin{aligned} R^2 &= .991 \\ SE &= .0003 \\ DW &= 1.83 \end{aligned}$$

The large coefficient for lagged dependent variable reflects slow speed of adjustment in the housing market. The final term reflects the impact of changes in population relative to real income. It should be possible to improve on this equation by exploiting the relation between YRENT and personal consumption of housing services. But to obtain the latter variable in current dollars, we would need the corresponding implicit deflator. However, we have deliberately chosen to minimize the number of deflators appearing in the model, a matter discussed further in Chapter X.*

The equation for NETINT arose from the following model. Let D^* be the desired amount of debt by businesses. Assume that D^* is determined by

$$D^* = a + b \text{ RCB} + c \text{ YGPPI},$$

where b should be negative to reflect the impact of debt costs and c should be positive to capture the influence of the level of activity.* If actual debt, D , adjusts to D^* according to

$$D - D(t-1) = \gamma [D^* - D(t-1)]$$

we can substitute and obtain

$$D = a\gamma + b\gamma \text{ RCB} + c\gamma \text{ YGPPI} + (1-\gamma)D(t-1)$$

*The specifications of equations (7.8) and (7.9) were strongly influenced by unpublished work of S. J. Turnovsky.

**In reality the desired quantity of debt will depend on interest rates, costs of equity capital, the amount and composition of business assets, tax deductibility and non-interest terms (maturity, repayment provisions, etc.). The strategy of model building adopted here obliges us to compress this complicated aspect of reality to achieve our goals in economical fashion.

If NETINT is assumed equal to RCB times D, this equation can be multiplied by RCB to obtain

$$\text{NETINT} = a\gamma\text{RCB} + b\gamma(\text{RCB})^2 + c\gamma(\text{RCB} - \text{YGPPI}) + (1-\gamma)[\text{RCB} - \text{NETINT}(t-1) / \text{RCB}(t-1)].$$

Unfortunately, estimates of this final equation were rather bad. Neither of the first two terms was ever near statistical significance. The best equation obtainable from this framework was the following:

$$(7.10) \quad \text{NETINT} = \underset{(1.67)}{.0002394(\text{RCB YGPPI})} + \underset{(38.2)}{.9739[\text{RCB NETINT}(\tau-1)/\text{RCB}(\tau-1)]}$$

$$\begin{aligned} R^2 &= .995 \\ SE &= .458 \\ DW &= 1.03 \end{aligned}$$

Clearly the lagged dependent variable is doing all the work since its t-statistic is large and so is the R^2 for the equation, while the first coefficient is not significant at the 10% level. This term is retained in the equation, however, to avoid the absurd implication that the level of net business interest payments is unrelated to economic activity. The Durbin-Watson statistic clearly signals mis-specification.

BDIV must be determined next. Two identities are used:

$$(7.11) \quad \text{BCP} = \text{BPBT} - \text{BCIVA}, \text{ and}$$

$$(7.12) \quad \text{BCPT} = \text{BCP} * \text{BCPTRT}$$

Dividends are logically a function of $(BCP - BCPT)$, after-tax profits.

Thus, behavioral equations for BCIVA and for BDIV are required.

Inventory Valuation Adjustment

BCIVA is given by the following expression, where H is the stock of business inventories, and PH is a price index constructed by the Commerce Department especially for this purpose (See Kuh, Brookings volume):

$$BCIVA = - [PH - PH(t-1)]H.$$

Notice that PH is an index, not an implicit deflator. Further, we have only STBIN as a measure of inventory stocks less an unknown constant. Using PGPP as a proxy for the true PH, we obtained the best of a rather horrid series of alternative BCIVA equations:

$$(7.13) \quad BCIVA = \underset{(1.08)}{-.5927} [PGPP - PGPP(t-1)] - \underset{(.537)}{.0013} [STBIN(PGPP - PGPP(t-1))] + \underset{(4.19)}{.5389} BCIVA(t-1)$$

$$R^2 = .422$$

$$SE = .859$$

$$DW = 2.17$$

Clearly this is flimsy. The lagged dependent variable, which has no theoretical justification, is the only source of statistical explanatory power. The other two coefficients have the expected sign, but their variables contribute almost nothing to the regression. We toyed with the idea of simply treating BCIVA as exogenous, since we were unable to obtain a satisfactory equation for it. However, BCIVA is logically endogenous and it may be important to capture the attendant decline in BCIVA in simulations that involve considerable inflation.

Dividends

The dividend equation follows directly from the work of Lintner (1966) which indicates that businesses smooth dividend payments, relating them to some measure of normal profits rather than to current profits. Experimentation with a cash flow variable led to results that were inferior to those based on profits alone. The equation selected for BDIV was

$$(7.14) \quad \text{BDIV} = \underset{(2.05)}{.0313} (\text{BCP}-\text{BCPT}) + \underset{(27.6)}{.9443} \text{BDIV}(t-1)$$

$$\begin{array}{rcl} R^2 & = & .992 \\ SE & = & .334 \\ DW & = & 1.51 \end{array}$$

A constant term, insignificant at the 10% level was deleted from the equation. An increase in after-tax profits would, according to (7.14), raise current quarter dividends by only 3¢. The equilibrium increase in dividends is 56¢, a number that agrees with most other studies. The implied mean lag of equation (7.14) is 17 quarters, which seems excessive.*

c. Personal Income, Taxes and Transfers

The identity relating personal income to YNIP is the following:

$$(7.15) \quad \text{YPERS} = \text{YNIP} + \text{YGGPI} - \text{BPBT} + \text{BDIV} - \text{WSACCR} + \text{BTRF} \\ + \text{GEINT} + \text{GETRFP} - \text{GRFICA} + \text{YCINT}.$$

In order to obtain personal income from this sector, we need behavioral equations for the last four quantities in (7.15).

* The Department of Commerce equation (See Popkin, 1965) for different time periods is quite similar, with lagged dividends having a coefficient of .897.

Government Interest Payments

Our equation for GEINT follows the form used by Kuh (1965). To a first approximation, the change in GEINT is given by

$$GEINT-GEINT(t-1) = r[GDBT-GDBT(t-1)+GDBT[r-r(t-1)]],$$

where r is the relevant interest rate. We tried both RCB and RCP as proxies for r and the shorter rate worked better:

$$(7.16) \quad [GEINT-GEINT(t-1)] = .0849 + .0053 [RCP(GDBT-GDBT(t-1))] \\ (4.29) \quad (1.51) \\ + .0011[GDBT(RCP(t-1))] \quad R^2 = .245 \\ (3.22) \quad SE = .141 \\ DW = 1.51$$

The total variance explained is not as bad as the R^2 indicates, of course, since the dependent variable is a first difference. The second coefficient is defensible only on a priori and not statistical grounds; the equation leaves much to be desired.

Government Transfer Payments

Government transfer payments are determined by the following equation:

$$(7.17) \quad GETRFP = 6.1003 + .0036(LRPOP \text{ TVMOA}) - .6996 \text{ LUT} + .0755(\text{TIME LUT}) \\ (2.06) \quad (8.36) \quad (1.08) \quad (5.75) \\ R^2 = .959 \\ SE = 2.03 \\ DW = .323$$

The second term captures social security payments. In the absence of any sensible time series for unemployment compensation benefit schedules, the last two terms were inserted to pick up unemployment compensation payments. It did not seem especially useful to delete the insignificant unemployment term and force the trend on benefit schedules through the origin.

Because of the trend term, this equation should not be extrapolated much beyond the period of fit. Trend terms can play a valid role in econometric applications, but since they nearly always drive an equation to plus or minus infinity, their use must be restricted accordingly.

Besides social security and unemployment compensation, there are a variety of other types of transfer payments made by governments, and these are picked up by the constant term.

The low Durbin-Watson of (7.17) rather clearly indicates misspecification. Our (untestable) surmise is that what is causing most of the trouble is the lack of a series for unemployment benefit rates. But the equation does "explain" the data fairly well.

Contributions for Social Insurance

The following equation is used to explain contributions for social insurance:

$$(7.18) \text{ GRFICA} = 3.2580 + .0023 [\text{TXRSS}(\text{YPRCE} + \text{YPTOT})\text{TMXEPE}] + .0106(\text{TXRUB} \text{ YPRCE})$$

(13.10) (2.35)
(21.3)

$$\begin{aligned} R^2 &= .994 \\ SE &= .738 \\ DW &= .155 \end{aligned}$$

The major social insurance programs are social security (OASI) and unemployment compensation; contributions to the remaining small programs are estimated by the constant. The second coefficient is a crude measure of OASI-taxable earnings times the tax rate; the influence of $TMXEPE$ would be exceedingly difficult to incorporate correctly. That this term represents a mis-specification is clearly indicated by the miniscule

Durbin-Watson statistic. But (7.18) explains the data well, and it is the best equation we could devise. Were it possible to separate OASI contributions from the rest of GRFICA, our predictions of total contributions would improve, but published data do not permit GRFICA to be split into its components.

Interest Paid by Consumers

The model underlying the YCINT equation is essentially identical to the one that gives rise to the NETINT equation. YDPI replaces YGPPI as the logical activity variable. In contrast with NETINT, the equation suggested by the theoretical development worked quite well here, with all terms significant and of the expected signs:

$$(7.19) \quad YCINT = .4165 \text{ RCB} - .1714(\text{RCB})^2 + .0021(\text{RCB YDPI}) + .7403[YCINT(t-1)]$$

(3.89) (3.35) (2.60) (6.65)

$$\text{RCB/RCB}(t-1)].$$

$$\begin{aligned} R^2 &= .992 \\ SE &= .265 \\ DW &= .763 \end{aligned}$$

Other than the low Durbin-Watson statistic, this equation is quite satisfactory. Relations between purchases of consumer durables, consumer debt, and YCINT have been ignored, for the usual reason that to do so would have required estimation of an equation for consumer durables implicit deflator.

Personal tax and non-tax payments must now be derived in order to obtain disposable income, since

$$(7.20) \quad YDPI = YPERS - GRPTX.$$

Personal Tax and Non-Tax Payments

Taxes are logically a function of the level of income, its distribution, and tax rates. The equation chosen to summarize these complex relationships was

$$\begin{aligned} (7.21) \quad \log(\text{GRPTX } 1000./\text{LTPOP}) &= -5.0779 + .4970 \log(\text{CGBTRT}) + 1.4925 \\ &\quad (24.4) \quad (8.71) \quad (40.7) \\ &\quad \log(\text{YPERS } 1000./\text{LTPOP}) \\ R^2 &= .987 \\ SE &= .0239 \\ \text{Antilog SE} &= 1.0241 \\ DW &= .838 \end{aligned}$$

The progressive nature of the tax structure is clearly indicated, as the last coefficient is significantly different from 1.0 at the 1% significance level. The low Durbin-Watson rather clearly signals mis-specification, but this equation seems adequate for our purposes.

SIMULATION RESULTS

The model simulated for this sector consisted of equations (7.2) - (7.6), and (7.8) - (7.21). Since YPRCE is identically determined by quantities exogenous to this sector, there was no sensible way for it to be an endogenous variable in this sector's runs.

The results of a dynamic simulation over the period 1954I - 1967IV are summarized in Table VII .2. The mean values shown there may help give some idea of the relative importance of the unfamiliar variables discussed in this chapter.

In terms of relative RMS errors, the two outstanding failures are the weak NETINT and BCIVA equations. Fairly large relative errors are made in

Table VII .2

Income Sector Simulation Results: 1954I - 1967IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a PCT. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
YNIP	394.56	1.90	.48	-1.23	-6.63***
BCCA	46.48	.98	2.10	.55	5.18***
BIBT	47.81	.94	1.95	.55	5.32***
BTRF	2.01	.07	3.50	- .032	-4.07***
BPBT	56.27	2.86	5.10	- .85	-2.29***
YPTOT	49.29	1.16	2.40	.19	1.25
YRENT	16.50	.44	2.70	- .27	-5.92***
NETINT	11.11	1.79	16.10	- .29	-1.26
BCP	57.14	2.95	5.20	- .91	-2.44**
BCIVA	- .87	.97	111.5	.06	.46
BCPT	24.94	1.28	5.10	- .52	-3.33***
BDIV	14.67	1.11	7.60	- .79	-8.13***
YPERS	429.18	3.80	.89	- .63	-1.26
GEINT	7.62	.63	8.29	.55	13.57***
GETRFP	28.55	1.96	6.80	.01	.04
GRFICA	22.24	.74	3.30	- .12	-1.26
YCINT	7.94	.65	8.20	- .14	-1.70*
YDPI	376.03	3.44	.91	- .54	-1.26
GRPTX	53.21	1.58	3.00	- .016	- .85

* Significant at 10%
 ** Significant at 5%
 *** Significant at 1%

TABLE VII .3

Income Sector Simulation Results: 1968I - 1968IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
YNIP	619.23	1.76	.30	1.43	2.21
BCCA	73.25	.43	.80	.18	.83
BIBT	77.93	2.12	3.00	-1.71	-2.26
BTRF	3.40	.125	4.00	.118	6.30***
BPBT	87.90	.44	.50	- .11	- .92
YPTOT	63.75	2.30	3.80	2.04	3.41**
YRENT	21.23	.22	1.00	.18	2.54*
NETINT	27.98	.87	3.00	- .69	-2.25
BCP	91.15	2.19	2.30	- .80	- .79
BCIVA	-3.25	1.76	48.10	.69	.82
BCPT	41.06	1.14	2.70	- .75	-1.55
BDIV	23.12	1.18	5.10	-1.16	-10.12***
YPERS	687.93	4.86	.70	-4.75	-6.15***
GEINT	11.88	.48	4.04	- .34	-1.73
GETRFP	55.75	6.04	11.70	-5.94	-9.52***
GRFICA	47.03	1.50	3.20	-1.49	-13.55***
YCINT	14.20	.39	2.60	- .36	- 4.24**
YDPI	590.02	3.32	.60	.71	.43
GRPTX	97.75	5.35	6.00	-5.32	-17.01***

* Significant at 10%
 ** Significant at 5%
 *** Significant at 1%

predicting corporate profits. This is not uncommon in macro-models, since corporate profits are most often obtained as a residual. The subtraction of two nearly equal numbers magnifies relative errors. Even though we might have been able to obtain better predictions of BPBT by another approach, corporate profits are logically a residual, and we retain the standard specification. The weakness of the BDIV equation stems in part from the errors made in RCP and BCPT, but, as the t-test indicates, the possibility of mis-specification should not be discounted. This model systematically underestimates dividends. Finally, it is clear that the GEINT equation, written in terms of the change in GEINT, goes off the track, probably because the errors cumulate over time. We said at the outset of the chapter that the main reason for this sector is to predict YDPI. That job is done well in the sample period.

A dynamic simulation was also run for the four quarters of 1968; the results are summarized in Table VII .3. In terms of relative RMS error, BCIVA still stands out. It is encouraging that the relative RMS error of this equation is much less in 1968, with a brisk inflation, than in the earlier sample period. This is just what we had hoped would happen.

NETINT is also much better predicted here than in the sample, as are business profits, YRENT, and GEINT. On the other side of the ledger, YCINT, BIBT, GRPTX, and GETRFP are not estimated as well in 1968. The last three problems may well reflect structural changes not captured by the equations. Forecasts of YPERS, GRFICA, GETRFP, and GRPTX are badly biased, probably for the same reason. But still, the key variable YDPI is forecast remarkably well. Indeed, the relative RMS error is lower in 1968 than in the sample period.

CHAPTER VII Income Distribution

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CHAPTER VIII

Employment and Unemployment

Of all the policy questions that macromodels are concerned with, matters relating to employment and unemployment have the highest priority. In this chapter we shall discuss the labor sector equations that generate employment, labor force, and unemployment. As matters have turned out, the first two of these can be quite sensibly explained, while the third, which is the difference between them, is subject to large percentage errors.

THE DATA

The variables appearing in the labor sector are described in Table VIII.1. All data except population and government employment are seasonally adjusted. The primary labor force and primary population is defined to include males aged 20 and over. The secondary labor force and secondary population is composed of males aged 16-19 and all females. These correspond fairly well to accepted categories except that males aged 65 and over (or perhaps 55 and over) are normally excluded from the primary labor force. It proved impossible to construct consistent quarterly labor force and population series that conformed to this definition, so older males were retained in the primary labor force.

The first six series in Table VIII.1 are quarterly averages of seasonally adjusted monthly figures taken from Employment and Earnings (U.S. Dept. of Labor - Bureau of Labor Statistics). LPEHR is an unpublished series compiled by the Bureau of Labor Statistics. The quantity LWHKR was calculated according to

$$(8.1) \quad LWKHR = LPEHR / (52 \text{ LEPVT}).$$

Gross private product in constant dollars was taken from the Survey of Current Business. Government civilian and military employment were taken from Employment and Earnings; quarterly averages of monthly figures were used. Data for LPPOP and LSPOP were taken from Employment and Earnings for 1964 and later years; figures for earlier periods were furnished by the Bureau of Labor Statistics. Both series refer to the total non-institutional population.

THE EQUATIONS

Behavioral equations have been estimated for LCP, LCS, LPEHR, and LWKHR. The following identities are employed to obtain the other endogenous variables:

$$(8.2) \quad LEPVT = LPEHR / (52 LWKHR).$$

$$(8.3) \quad LET = LEPVT + LEGC + LEGM.$$

$$(8.4) \quad LUT = (LCP + LCS) = (LEPVT + LEGC)$$

$$(8.5) \quad LUR = LUT / (LCP + LCS).$$

The first of these comes from equation (8.1), the identity used to compute WKHRS. The second equation merely adds the various categories of employment. Total unemployment is computed in equation (8.4) by subtracting civilian employment from the civilian labor force. Finally, equation (8.5) is an obvious identity that determines the unemployment rate.

TableV111.1

Variables Appearing in the Labor Sector

(Endogenous: Determined in Labor Sector)

LCP	Primary civilian labor force, millions of persons.
LCS	Secondary civilian labor force, millions of persons.
LEPVT	Total private employment, millions of persons.
LET	Total employment, millions of persons.
LUR	Unemployment rate: unemployment as a fraction of the civilian labor force.
LUT	Total unemployment, millions of persons.
LPEHR	Total man-hours worked in the private economy, millions of hours at annual rates.
LWKHR	Average hours worked per employee per week in the private economy.

(Endogenous: Determined Elsewhere in Model)

YGPP	Gross private product, billions of 1958 dollars at annual rates.
------	--

(Exogenous)

LEGC	Government civilian employment, millions of persons.
LEGM	Government military employment, millions of persons.
LPPOP	Primary population, millions of persons.
LSPOP	Secondary population, millions of persons.
TIME	Time trend: equal to 1.0 in 1954I, increases by 1.0 each quarter.

All National Income Figures are seasonally adjusted at Annual Rates

We shall first discuss the determination of LCP and LCS.

Numerous studies have come to the conclusion that the tighter is the labor market, the more persons seek employment.* In particular, persons other than prime-aged males enter the labor force. How labor market tightness should be measured is a hard empirical question. Various functions of the unemployment rate were tried, but with absolutely no success. The best measure was the ratio of civilian employment to the civilian population, which will be written as ER. This quantity is defined by the following equation:

$$(8.6) \quad ER = (LET - LEGM) / (LPPOP + LSPOP - LEGM).$$

ER as defined by (8.6) was slightly superior to the same variable without the subtraction of LEGM from numerator and denominator. Changes in ER would not be expected to affect participation immediately, so all equations included geometric distributed lags.

Secondary labor force participation

Separate equations were estimated for primary and secondary labor force participation rates. Experiments with models involving only one equation for overall participation were attempted, but the results were inferior to those presented here. Our best equation for secondary participation is the following:

* See Jacob Mincer, "Labor-Force Participation and Unemployment. A review of Recent Evidence", in Gordon & Gordon, eds; 1966:73-134; Alfred Tella, "The Relation of Labor Force to Employment", Inds.Lab.Rel.Rev., Apr.64, 17,454-69; & "Labor Force Sensitivity to Employment by Age, Sex," Indus.Rel., Feb.1965,4,67-83; K. Strand and T. Dernberg, "Cyclical Variation in Civilian Labor Force Participation", Rev. Econ. Stat., Nov. 64, 46, 378-91.

$$\begin{aligned}
 (8.7) \quad & \text{LCS/LSPOP} + .3797 + .5107 [\text{LCS}(t-1)/\text{LSPOP}(t-1)] \\
 & \quad (7.70) \quad (6.61) \\
 & + .0004319 \text{ TIME} - .1157 (1/ER) \\
 & \quad (6.21) \quad (6.44)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .956 \\
 SE &= .0031 \\
 DW &= 1.50
 \end{aligned}$$

The positive time trend reflects a secular tendency towards increased participation of women in the labor force. Equation (8.7) estimates the secular increase in the participation ratio to be about .54% per year. This trend cannot persist indefinitely since the participation ratio cannot exceed unity. Hence (8.7) should not be used much beyond the period of fit.

Equation (8.7) indicates a swift response of the secondary labor force to changes in employment opportunities; the mean lag is just over one quarter. The initial response to a fall in $(1/ER)$ of .10 is an increase in the secondary participation rate of .012, while the equilibrium increase is .024.

Primary Labor Force Participation

An equation of the same form as (8.7) also performed best for primary labor force participation:

$$\begin{aligned}
 (8.8) \quad & \text{LCP/LPPOP} = .3594 + .7809 [\text{LCP}(t-1)/\text{LPPOP}(t-1)] \\
 & \quad (2.75) \quad (8.20) \\
 & - .0002731 \text{ TIME} - .0949 (1/ER) \\
 & \quad (2.36) \quad (2.32)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .930 \\
 SE &= .0069 \\
 DW &= 1.68
 \end{aligned}$$

The negative time trend shows the tendency towards earlier retirement; the estimated secular decline in the primary labor force participation ratio is about .495% per year. Again, this trend cannot be expected to hold much beyond the period of fit. This trend has the same magnitude and opposite sign to the secondary labor force participation equation, so that overall labor force participation is more stable than either major constituent.

Equation (8.8) indicates a sluggish response of the primary labor force to changes in employment conditions; the mean lag is about 3.65 quarters.* The initial response to a fall in $(1/ER)$ of .10 is an increase in the primary participation rate of .009, while the equilibrium increase is .043. Thus, while the initial response is just slightly smaller than for secondary participation the equilibrium response is greater. One can rationalize a small initial primary response and sluggish adjustment on the grounds that all potential workers in this group are almost always seeking employment, but we are at a loss to explain the larger equilibrium response of the primary participation ratio to changes in ER . This latter result was obtained for almost all alternative participation rate equations that were estimated.**

* Equations (8.7) and (8.8) performed just slightly better than equations in which ER replaced $(1/ER)$, indicating that the impact of changes in ER is less the larger is ER .

** The equilibrium response may be biased by the inclusion of males 55 years and older.

Manhours Demand

We now turn to the determination of total private employment. The fundamental labor input into the production process is manhours, measured by LPEHR. This relationship follows closely along lines established by Nield (1963), Brechling (1965), Eckstein (1964), Kuh (1965) and Black and Russell (1969). ^{(Wilson (1966))} [§] It is reasonable to assume that changes in output will not be immediately reflected in changes in manhours worked, since non-production workers, who are assumed to work a fixed number of hours per week, will not be hired or fired in response to changes in demand that their employers believe might be transitory. The Cobb-Douglas production function has had wide popularity in econometric work. We have adopted it here as the simplest equation form with acceptable economic characteristics over which a linear production function does not possess.* This form suggests that the demand function for man-hours should be linear in the logarithms. Various functions of this sort were tried. In none were capital stock variables significant, even though the logic of the Cobb-Douglas suggests they should be. There are two obvious explanations for this: the capital stock series are excessively crude, and the Cobb-Douglas exaggerates the short-run substitution possibilities between capital and labor. Our demand function for man-hours finally turned out to be:

* See Marc Nerlove Estimation and Identification of Cobb-Douglas Production Functions, Rand McNally & Co. - Chicago, 1965.

$$\begin{aligned}
 (8.9) \quad \log(\text{LPEHR}) &= 3.193 - .0026 \text{ TIME} + .3599 \log(\text{YGPP}) \\
 &\quad (7.02) \quad (8.50) \quad (9.44) \\
 &\quad + .5471 \log(\text{LPEHR}(-1)) \\
 &\quad (10.5)
 \end{aligned}$$

$R^2 = .9785$
 $\text{SE} = .0058$
 $\text{Antilog SE} = 1.0058$
 $\text{DW} = 1.34$

The negative time trend reflects both technical progress and secular increase in capital per worker. The mean lag in (8.9) is 1.2 periods, implying rather rapid adjustment of man-hours to changes in output. The short-run elasticity of man-hour demand with respect to output is .36, while the long-run elasticity is .795. The fact that the long-run elasticity is less than one suggests increasing returns to scale in the underlying aggregate production function although since it is subject to sampling error, this conclusion is highly uncertain. An alternative interpretation is, of course, that we have failed to include a good measure of capital and have, therefore, missed much of the impact of capital deepening. In any case, equation (8.9) is the best we have been able to devise, resembles other such aggregate relations, is the best we have been able to devise, explains the data quite well, and the coefficient estimates are not totally absurd.*

* Black and Russell (1969) obtain results with a significant capital stock variable for the 1947-65 period of fit. When they use more recent data (1955-65) which are comparable to ours in every respect: "The poor quality of the capital stock data, and perhaps underutilization of capital, presumably contributed to this result. In addition, for purposes of computation of potential GNP, the capital stock is fixed in the short run. Their SE is .0054 and reaction coefficient is .50, while their short-run output elasticity of manhour demand is .39.

Weekly Hours

Some of the information in equation (8.9) will be utilized in the estimated equation for LWKHR. Let M be employment, H be average hours per employee, Y be gross private product and set t equal to TIME.

The model underlying (8.9) may be compactly written as

$$(8.10) \quad [MH] = AY^{\alpha\gamma} e^{-\beta\gamma t} [M(t-1)]^{1-\gamma}$$

Suppose that the demand for employees is given by the following quite similar model:

$$(8.11) \quad M = BY^{a\delta} e^{-b\delta t} M(t-1)^{1-\delta}$$

Since one would expect employment to adjust more slowly than man-hours (people work overtime when demand rises suddenly), γ should be larger than δ . Notice that γ and δ must fall between zero and one. Similarly, since the average work week has been secularly falling, the demand for employees must be dropping off more rapidly than the demand for man-hours; thus β should be larger than b . Finally, it does not seem unreasonable for α to be equal to a , though a-priori this seems the weakest of the three assertions.

Dividing the second equation above by the first and taking logarithms, we obtain

$$(8.12) \quad \log[H/H(t-1)] = \log (A/B) + (\alpha\gamma - a\delta) \log (y) + (b\delta - \beta\gamma) t \\ + (-\gamma) \log [H(t-1)] + (\delta - \gamma) \log [M(t-1)].$$

The last three terms should have negative coefficients, according to our a priori beliefs, while if α is not much larger than α , the coefficient of $\log(Y)$ will be positive. When this model was estimated, we obtained

$$\begin{aligned}
 (8.13) \quad \log \text{LWKHR}/\text{LWKHR}(t-1) &= 1.306 + .1587 \log(\text{YGPP}) \\
 &\quad (4.87) \quad (6.08) \\
 &- .0018 \text{TIME} - .5342 \log \text{LWKHR}(t-1) - .0604 \log \text{LEPVT}(t-1) \\
 &\quad (5.56) \quad (4.45) \quad (.908) \\
 R^2 &= .4889 \\
 SE &= .0039 \\
 SE &= 1.0039 \\
 \text{Antilog DW} &= .185
 \end{aligned}$$

All coefficients have the expected signs, and all are significant except the last. The coefficient of $\log \text{LWKHR}(t-1)$ gives an estimate of $\gamma = .5342$, as against $\gamma = .4529$ estimated by (8.9).

We imposed the latter estimate on (8.13) and re-estimated obtaining

$$\begin{aligned}
 (8.14) \quad [\log(\text{LWKHR}) - .5471 \log \text{LWKHR}(t-1)] &= 1.148 \\
 &\quad (8.75) \\
 + .1540 \log(\text{YGPP}) - .0016 \text{TIME} - .0897 \log[\text{LEPVT}(t-1)] \\
 &\quad (6.16) \quad (9.69) \quad (1.78) \\
 R^2 &= .7912 \\
 SE &= .0039 \\
 \text{Antilog SE} &= 1.0039 \\
 \text{DW} &= 2.01
 \end{aligned}$$

The F-statistic for the restriction imposed on (8.14) is .462, with one and fifty-one degrees of freedom. We cannot reject the hypothesis $\gamma = .4529$ at any reasonable significance level.

If a were equal to α , the absolute value of the ratio of .1540 to $-.0897$ would be an estimate of the common value of these parameters. This figure is 1.72, decidedly larger than the estimate of α of .795 in equation (8.9). We reestimated (8 .14) imposing the constraint that $\alpha = a = .795$, but the hypothesis was rejected at the 1% level of significance. This suggests that a is smaller than α .

Equation (8 .14) is the preferred LWKHR equation, even though the coefficient of lagged employment is insignificant at the 5% significance level. We have made this decision because the coefficient has the right sign, it is significant at the 10% significance level, and it is a basic part of the theoretical model.

The labor sector thus consists of the identities (8 .2) - (8 .6), of which only four are essential since the equation for ER merely simplifies presentation, and the four stochastic equations (8 .7) - (8 .9) and (8.14).

SIMULATION RESULTS

A dynamic simulation of the labor sector was performed over the period of fit, 1954I - 1967IV, and a summary of the results is shown in Table VIII.2. By looking at the root-mean-squared errors, it is clear that the participation equations contain some weaknesses, but the errors fortunately are relatively small. Larger relative errors crop up in LUT and LUR. It is clear why this happens. The labor force is forecast with modest errors, as is civilian employment. Since these two quantities are nearly equal, when we subtract to obtain unemployment the small relative

errors in employment and labor force become large relative to errors in LUT. These then produce large relative errors in LUR. Thus, either far superior LCS and LCP equations or an entirely different approach would be required to noticeably improve our ability to track LUT and LUR.

The t-tests indicate a systematic tendency to over-estimate average weekly hours, leading to a tendency to underestimate employment. This part of the model does go off the track, but the RMS errors seem too small for this to be a cause of great alarm. We also persistently over-estimate the secondary labor force.

An ex post forecast of the endogenous variables in 1968, given YGPP and the exogenous quantities was also calculated. The results are shown in Table VII.3. The RMS errors follow basically the pattern in Table VII.2, with the labor force equations being the weakest stochastic equations and the LUR and LUT predictions being generally wide of the mark.

The t-statistics suggest that some specification problems still persist. Slight under-estimation of man-hour demand and slight over-estimation of average weekly hours lead to a systematic but slight underestimation of employment. As in the earlier simulation, the primary labor force is overestimated and the secondary labor force is underestimated. On balance, the total labor force is overestimated, so that consequently both LUR and LUT are overstated, even though the t-test suggests there is no bias.

Table VIII.2

Labor Sector Simulation Results: 1954I - 1967IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
LPEHR	129000.	1029	.79	77.1	.5569
LWKHR	40.83	.169	.41	.1031	5.6213***
LEPVT	60.79	.348	.57	-.1188	-2.70***
LET	69.45	.348	.501	-.1188	-2.70***
LCP	43.62	.742	1.701	.0086	1.194
LCS	26.47	.423	1.59	.250	5.4359***
LUT	3.56	.854	24.00	-.0421	-.3734
LUR	.051	.011	21.58	-.00052	-.3508

*Significant at 10%

**Significant at 5%

***Significant at 1%

Table VIII.3

Labor Sector Simulation Results: 1968I - 1968IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
LPEHR	140600.	226.	.161	-46.32	-.362
LWKHR	40.20	.075	.186	.026	.0641
LEPVT	67.27	.079	.118	-.066	-2.634*
LET	79.47	.079	.0998	-.066	-2.634*
LCP	45.85	.535	1.17	.515	6.149***
LCS	32.90	.306	.93	-.289	-5.193**
LUT	2.81	.329	11.7	.308	4.716**
LUR	.036	.00382	10.6	.00369	7.449***

*Significant at 10%

**Significant at 5%

***Significant at 1%

CHAPTER VIII Employment and Unemployment

Labor Force Participation

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Manhour Demand

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Average Weekly Hours

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CHAPTER VIII (cont'd)

Labor Force Participation and Manhour Demand

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The Cobb-Douglas Production Function

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Chapter IX

Wages and Prices

In this sector, equations for the average compensation per man-hour in the private economy will be derived. Based largely on this wage construction, four endogenous implicit price deflator have also been estimated. Our approach to the wage-price sector began rather conventionally, but we have ended up adopting a somewhat novel wage equation, since standard wage equations have weak theoretical and statistical foundations.

We shall first discuss the rather straightforward data used in this sector, which is followed by the model of wage determination and the price equations. The chapter concludes with a discussion of two simulation experiments with this sector.

THE DATA

The variables appearing in this sector are listed in Table X.1. The first of these, LPEHR, was obtained by solving the estimated private manhour demand function, equation (9.9), for its equilibrium value. The formula used to obtain LPEHR is

$$(9.1) \quad \log(\text{LPEHR}) = 7.05 - .00575 * \text{TIME} + .795 \log(\text{YGPP})$$

WRPVT was computed according to

$$(9.2) \quad \text{WRPVT} = (\text{YPRCE } 1000) / \text{PEHRS},$$

Table IX.1

Variables used in Wage-Price Sector

(Endogenous Variables - Determined in Wage-Price Sector)

LPEHR	Equilibrium manhour demand in the private economy, millions of manhours at annual rates.
WRPVT	Average compensation per manhour in the private economy, current dollars per manhour.
PGPP	Implicit price deflator for gross private product, 1957-59 = 100.
PGGP	Implicit price deflator for gross government product, 1957-59 = 100.
PCPTOT	Implicit price deflator for total consumption spending, 1957-59 = 100.
PGGD	Implicit price deflator for government purchases other than compensation of employees, 1957-59 = 100.

(Endogenous Variables - Determined Elsewhere in the Model)

YGPP	Gross private product, billions of constant dollars.
YGPPI	Gross private product, billions of current dollars.
CPTOTD	Total personal consumption expenditure, billions of constant dollars.
GGD	Government purchases of goods and services other than compensation of employees, billions of constant dollars.
PEHRS	Total manhours worked in the private economy, millions of manhours at annual rates.
WKHRS	Average hours worked per employee per week in the private economy.

(Exogenous Variables)

LEGM	Government military employment, millions of persons.
LEGO	Total government employment, millions of persons.
TIME	Time trend, equal to 1.0 in 1954I, rises by 1.0 each quarter.

where YPRCE is private compensation of employees in billions of current dollars, equal to total compensation of employees from the Survey of Current Business minus gross government product (compensation of government employees), and PEHRS is an unpublished series furnished by the Bureau of Labor Statistics and described in Table IX.1. The next three endogenous variables determined in this sector are taken directly from the Survey of Current Business. The last PGGD, was computed as follows. Total government purchases of goods and services minus government compensation of employees equals GGDI when both are in current dollars from the Survey. This difference equals GGD when both are the Survey's constant dollar estimates. Then PGGD is simply one hundred times the ratio of GGDI to GGD.

The first three endogenous variables used here but determined elsewhere were taken directly from the Survey, and the computation of the fourth was just described. PEHRS was also described above, and WKHRS was computed according to

$$(9.3) \quad WKHRS = PEHRS / (LEPVT \ 52),$$

where LEPVT is the quarterly average of seasonally-adjusted monthly estimates of total employment in the private sector taken from Employment and Earnings.

The first two exogenous variables listed are also quarterly averages of monthly figures from Employment and Earnings, and the third exogenous variable is self-explanatory, at least on a superficial level.

a. Wage Determination

The standard model of wage determination is based on the Phillips

Curve (Phillips, 1958) which says that the tighter are labor markets, as measured by (the reciprocal of) unemployment, the more rapidly wages rise. This treatment of wage adjustment has its origins in theoretical models of dynamic adjustment to excess demand. Recent investigators have greatly elaborated upon this formulation by allowing for lags in the rate of change of wages, the impact of consumer prices, and the effect of corporate profits, (Lipsey, 1960; Perry, 1966; Phillips, 1958). These last two factors enter positively, as might be supposed.

We experimented with variations on this theme. All were consistently out-performed by a model due to Kuh (1967).^{*} This approach follows standard price theory more closely by postulating that the equilibrium level of wages is proportional to the marginal value product of labor, which in turn varies with the average value product of labor. Taking the private sector as a whole, the average value product of labor is defined by

$$(9.4) \quad AVP = (PGPP \ YGPP)/PEHRS$$

This model permits the actual wage to adjust gradually to the equilibrium wage level. The equations, furthermore, are linear in the logarithms since this specification is appropriate to the commonly-encountered Cobb-Douglas production function.

Kuh found that when AVP is used in wage equations, neither measures of labor market tightness, such as the unemployment rate, nor the level of

* Dennis Sargan has also proposed a model of this nature in his "Wages and Prices in the United Kingdom: A Study in Econometric Methodology (P. E. Hart, et al, eds.) Econometric Analysis for National Economic Planning (London, 1964).

corporate profits are significant. We also found this to be true. Similarly, we found, as did Kuh, that consumer prices exerted only a transient (though statistically significant) influence upon wages. No measure of the rate of change of YGPP nor any measure of capacity utilization (such as WKHRS or the change in WKHRS) was statistically significant.

Kuh's basic formulation was modified by adopting an approach often used in studies of price determination. (See Schultze and Tryon, 1965; Eckstein and Fromm, 1968) and in the study of potential output (Black and Russell, 1969). Since labor is not hired or fired instantaneously whenever demand rises or falls, observed productivity is affected by the fact that actual manhours worked are rarely equal to equilibrium manhours, because labor inputs adjust with some lag to cyclical variation in output. Since labor and management are well aware of this fact, one might expect cyclical variations in AVP to have only a minor impact on wage determination. We consequently distinguish between the observed average value product of labor, AVP, and the normal or equilibrium average value product of labor, AVPE. We tried simple moving averages of past values of AVP as an approximation of AVPE, just as Kuh did, but this variable was outperformed consistently by the following construction:

$$(9.5) \quad AVPE = (PGPP \ YGPP) / LPEHR,$$

where $LPEHR$ is defined by (9.1).

Unexplained variance was about 20% smaller when the preferred variable was used. The version of the Phillips Curve presented in Kuh's

article had a much higher R^2 than the same formulation applied to these data, which have only six years in common and eleven disparate years. Private non-farm compensation per hour was the dependent variable in the Kuh article while here it is the same compensation definition but for the entire private sector.

The best equation obtained for WRPVT, and the one appearing in the model, is the following:

$$\begin{aligned}
 (9.6) \quad \log(\text{WRPVT}) = & - .3337 + .2136 [\log(\text{AVP}) - \log(\text{AVPE})] \\
 & (.65) \quad (3.06) \\
 & + .6846 \log(\text{AVPE}) + .3284 \log[\text{WRPVT}(t-1)] \\
 & (6.34) \quad (3.09) \\
 & + .5209 [\log(\text{PCPTOT}) - \log(\text{PCPTOT}(t-1))] \\
 & (2.02)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .005 \\
 \text{Antilog SE} &= 1.005 \\
 DW &= 1.72
 \end{aligned}$$

The mean lag of less than a month is much less than that found in most previous work, and at first glance appears to cast serious doubt on the equation's validity. Further reflection indicates that this suspicion is not well founded. For one thing, much non-union labor is included, and their wages adjust more rapidly than union wages under a regime of two or three year contracts. Second, the major determinant of wages - equilibrium labor productivity - is an extremely smooth series, so that while wage changes are themselves smooth and small, the adjustment speed is rapid.

The long-run elasticity of wages with respect to AVPE is equal to 1.02, which is essentially unity as it should.* Equation (9.6) clearly shows that the equilibrium value of AVP is more important than deviations from equilibrium as labor input is being adjusted. Consumer prices, measured here by PCPTOT, do exert an important transient influence on WRPVT, but the level of PCPTOT does not affect the equilibrium level of WRPVT. Not only is equation (9.6) more in line with neo-classical price theory than the usual approach, but it is statistically sounder as well. These results contrast with Phillips curve theory and imply that labor market conditions do not influence wages. We have been unable to find any convincing evidence for such effects in our data.

Perhaps Phillips Curve doctrine is more applicable to periods of massive unemployment or when unemployment fluctuates more widely than between 3.5% and 7.5%. The rather neoclassical approach of this paper is consistent with full employment behavior, a condition more nearly met historically since World War II than at any other time in our history.**

* This corresponds to a constant or slowly increasing wage share, the former in strict conformity to theory implicit in the Cobb-Douglas function, the latter with historical fact. Kuh found the same thing, although his reaction coefficients of .09 to .22 were much lower.

** Among the most interesting attempts to generalize and validate the Phillips Curve, see George de Menil and Jared J. Enzler, "Wages and Prices in the FRB-MIT-Penn Econometric Model", FRB-SSRC conference on "The Econometrics of Price Determination", Washington, D.C., October 30, 1970. Mimeographed, unpublished. They have divided the labor market into a unionized and non-unionized sector, and to treat explicitly contract negotiations, minimum wages, composition effects and overtime hours, among other refinements. Within our mandate of relative parsimony of form, variables and exposition, it is not possible to incorporate this work. How much of a gain it will be over the system presented here remains to be seen.

b. Price Equations

Price of Gross Government Product

The implicit deflator for gross government product is conceptually simple; it is just a weighted average of government wage rates. These should be related to the average wage rate paid in the private sector, possibly with a lag. If there has been a difference in the rate of growth of civilian and military wages, the civilian-military mix of government employment should affect the relation between the average government wage and the average private wage.

The best equation found for PGGP was the following:

$$\begin{aligned}
 (9.7) \quad PGGP &= 2.7873 + 13.2008 \text{ WRPVT} + .7664 \text{ PGGP}(t-1) \\
 &\quad (2.17) \quad (3.52) \quad (11.1) \\
 &\quad - 3.3965 [\text{WRPVT LEGM/LEGO}] \\
 &\quad (1.61)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .706 \\
 \text{Antilog SE} &= 2.030 \\
 DW &= 1.81
 \end{aligned}$$

Equation (9.7) has a mean lag of about 3.3 quarters, which seems reasonable. We retain the last term even though it is not significant at the 10% level because it correctly indicates the effect of military wages rising less rapidly than the wages paid civilian government employees.

Price of Gross Private Product

Econometric determination of the implicit deflator for gross private product is fairly standard. We assume that prices are basically determined as a markup on unit labor cost. This quantity is defined for the entire private sector as

$$(9.8) \quad ULC = (\text{WRPVT PEHRS})/\text{YGPP}$$

Following the studies cited above, we distinguish between observed unit labor cost, ULC, and normal or equilibrium unit labor cost, ULCE. We attempted to use moving averages of past values of ULC as a proxy for ULCE, but we obtained better results using equilibrium manhours in a similar fashion to that employed in the wage equation:

$$(9.9) \quad ULCE = (WRPVT \cdot LPEHR) / YGPP,$$

where ~~LPEHR~~ is defined by equation (9.1).

The log-linear functional form is convenient, although it is not implied by markup theories of pricing. Presumably prices adjust to costs with some sort of lag, and this factor was allowed for. Finally, the price level ought to depend on how "tight" product markets are, that is, on short-run pressures on currently-employed labor and capital. As a measure of these pressures, ~~we took the ratio of WKHRS to~~ equilibrium hours.* We assumed equilibrium hours to be a decaying exponential function of time over the sample period, an assumption consistent with the observed secular decline in average hours worked due in large measure to an increasing proportion of women in the labor force working part time. Thus both $\log(WKHRS)$ and TIME would be expected to have positive coefficients.

The equation for PGPP used in the model is consistent with this discussion and with earlier investigations:

* Overtime and short time are the most flexible controls available to business management faced with the need to manhour inputs to short run output changes. See C.C. Holt, F. Modigliani, J. F. Muth, and H. A. Simon, "Planning Production, Inventories and Work Force" (Englewood Cliffs, N.J.:Prentice-Hall, 1960), and Edwin Kuh, "Income Distribution and Employment Over the Business Cycle", The Brookings Quarterly Econometric Model of the United States, Duesenberry et al, Part IV, Chapter 8, pp.227-278, 1965, Rand McNally & Co., Chicago, North Holland Pub. Co., Amsterdam. Thus workhour changes are very closely related to shortrun variations in capacity utilization and

$$\begin{aligned}
 (9.10) \quad \log(\text{PGPP}) &= .2539 \log(\text{ULCE}) + .1094[\log(\text{ULC}) - \log(\text{ULCE})] \\
 &\quad (5.88) \qquad\qquad\qquad (6.34) \\
 &+ .1330 \log(\text{WKHRS}) + .0002117 \text{ TIME} \\
 &\quad (4.48) \qquad\qquad\qquad (2.66) \\
 &+ .7101 \log(\text{PGPP}(t-1)) \\
 &\quad (14.3)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .0021 \\
 \text{Antilog SE} &= 1.0021 \\
 DW &= 1.96
 \end{aligned}$$

This equation has a mean lag of 2.45 quarters, considerably longer than the mean lag of the wage equation. As most studies of price formation have found, we find that ULCE is more important than short-run deviations of observed ULC from this equilibrium value. The long run elasticity of price with respect to unit labor cost is .86, not much different from unity. We would expect the markup to be near unity, although changing intensity of competition and aggregation effects could readily enough explain the observed outcome.

The steady state properties of the wage-price sector are worth some comment. The steady-state ^{real} wage is approximately $\frac{W^*}{P^2} = A \cdot APE^*$ where APE^* is equilibrium ^{real private product per manhour or} average labor productivity, P^* is steady state price and W^* is steady state wage and A is all other effects, assumed constant. The steady state price is approximately $P^* = B \cdot W^* / APE^*$, where B is a constant analogous to A and all other variables have been defined, so that consistency requires $B = \frac{1}{A}$. Thus the model determines the real wage in a steady state, which in turn depends primarily on labor productivity. The absolute price and wage level will be determined elsewhere in the system, perhaps

by the quantity of money. This is ~~is~~ Keynesian-Classical full employment theory, which is not unsuited to the post-war U.S. economy and seems more consistent with the data than most alternative versions we have seen. Models are, after all historical representations: it would be most remarkable for this or any alternative model to capture the essence of any economy in all extreme conditions, however attractive this would be. The use of this model to simulate extreme behavior - at least so far as price/wage behavior is concerned - should be undertaken with the most extreme caution.

We now must determine the price deflators for total consumption and for government goods and services purchased from the private sector. As we have no industry data in this model, it is impossible to relate these prices to their fundamental determinants as in the case of $PCPTOT$ and $PCGP$. Instead, we assume that the rate of change of these deflators is straightforwardly related to the rate of change of the deflator for gross private product. Also, it is reasonable to assume that the more important these categories are in the total demand for private output, the more rapidly their prices will rise. A shift of demand to consumer goods, for instance, will generally force the prices of such goods up more rapidly than the average of private sector prices.

We used the change in the logarithm as an approximation to the percentage change of the various deflators, which is valid when the percentage changes are small. A variety of specifications were tried, most notably some involving the change in the logarithm of $WRPVT$. All of the latter failed.

Price of Consumption Outlays

The best equation for PCPTOT was:

$$\begin{aligned}
 (9.11) \quad [\Delta(\log PCPTOT)] &= -.0426 \\
 &\quad (2.02) \\
 &+ .7295[\Delta \log PGPP] \\
 &\quad (9.49) \\
 &+ .1298 \Delta \log PGPP(t-1) \\
 &\quad (1.85) \\
 &+ .0607 (CTOT/YGPP) \\
 &\quad (2.03)
 \end{aligned}
 \qquad
 \begin{aligned}
 R^2 &= .676 \\
 SE &= .0017 \\
 \text{Antilog SE} &= 1.0017 \\
 DW &= 1.79
 \end{aligned}$$

Notice that even though the standard error of (9.11) is comparable with that of (9.10), the R^2 is dramatically lower. Published price indices are largely trend, so it is not too difficult to obtain good fits to the level of prices. Explaining the rate of change of prices, however, is far more difficult, a point explored further in the next section. All terms are significant with the expected signs except for the lagged price change term. It was retained in the equation since it had a reasonable coefficient and because without it, the change in the logarithm of PGPP was not significant. The sum of the lagged and current coefficient equals .86, so that consumption prices are almost proportional to GNP prices in equilibrium.

Price of Government Product Less Employee Compensation

The equation that best explained changes in PGGD was considerably less satisfactory than (9.11):

$$(9.12) \quad \Delta \log PGGD = 1.4965 \log PGGP \\ (6.33)$$

$$\begin{aligned} R^2 &= .280 \\ SE &= .010 \\ \text{Antilog SE} &= 1.010 \\ DW &= 1.90 \end{aligned}$$

The Durbin-Watson statistic, somewhat surprisingly, suggests no serial correlation, given the poor fit and the extreme simplicity of the specification. Cynical readers of Joint Economic Committee reports on government procurement practices may be able to rationalize the large coefficient of the price change variable. The second term was retained because it had the right sign and because we felt that we could use all the explanatory power we could get.

SIMULATION RESULTS

Dynamic simulations were performed on the wage-price sector, consisting of the transformed equation (9.1) and behavioral equations (9.6), (9.7), and (9.10)-(9.12). (The definitions (9.4), (9.5), (9.8), and (9.9) were used only to simplify presentation)*. As $LPENR$ is a function only of quantities exogenous to the sector, its simulated values are identical to the true values and are hence not discussed. To consider the ability of this sector to forecast the rate of inflation, we added the following equation to the model:

$$(9.13) \quad IN = 400. \frac{\Delta PGGP}{PGGP(t-1)}$$

Thus IN is the annual rate of inflation expressed as a percentage.

*

Other interesting simultaneously determined price-wage studies are Dicks-Mireaux, "The Interrelationship Between Cost and Price Changes; 1946-1959: A study of Inflation in Post-war Britain". Oxford Economics Papers 13(1961), 267-292., and Dennis Sargan, op.cit).

Simulation results are mixed. The two central equations of this sector, the wage equation and the gross private product price deflator, have root mean square errors that are about 1% of the respective values for the mean of each dependent variable during the long simulation from 1954 through 1967, shown in Table IX.2.* Considering the high degree of interdependence between these two equations, this outcome is highly satisfactory. Less acceptable, however, are the large t-statistics which indicate that both equations tend to underpredict systematically. The 1968 simulations show the same bias which is statistically less significant. On balance these results are acceptable, usable within the context of this model, and comparable in most of their error properties to other macroeconomic models.

The unimportant government goods deflator PGGD has large errors, but these cannot seriously affect the overall quality of model behavior. The inflation measure, which estimates the rate of change of PGPP as opposed to its level, has a much larger relative error: this is a pervasive characteristic of econometric estimates. Bias in the measure, however, is negligible relative to its standard error.

*Phillips curve estimates were so poor that simulations based on them were too erratic to report here. When quite intricate Phillips including many plausible subsidiary variables were estimated for the FRB-MIT-Penn Model, superior estimates to our own were found. But the key difference was to use overlapping two-quarter differences for the dependent variable instead of one quarter independent changes. In our view there is inadequate statistical theoretical basis to justify this transformation. See George de Menil and Jared J. Enzler, (op.cit).

Simulations beyond the period of fit show a larger RMS for WRPVT and PCPTOT, the same for PGGP and much smaller for IN. The average bias is typically larger, but because there are so few observations, four in all, the tests suggest that the statistically significant bias is less. In no instances have the extrapolation period estimates diverged - to an extent that undermines conclusions reached on the basis of statistical criteria and the longperiod simulations.

TABLE IX.2

Wage-Price Sector Simulation Results

1954I - 1967 IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Error as a Pct. of Mean Value</u>	<u>Mean Error</u>	<u>t-test for Bias</u>	
WRPVT	2.01	.024	1.187	- .014	-5.478	***
PGPP	102.54	1.024	.999	- .721	-7.354	***
PGGP	111.44	1.096	.983	- .574	-4.559	***
PCPTOT	102.76	.710	.691	.047	.495	
PGGD	101.27	4.252	4.199	.801	5.275	***
IN	1.86	.821	44.140	- .080	-2.696	***

1968I - 1968IV

WRPVT	2.97	.058	1.960	- .054	-4.367	**
PGPP	118.90	1.050	.883	- .946	-3.596	**
PGGP	159.38	1.710	1.073	-1.319	-2.099	
PCPTOT	118.55	1.317	1.111	-1.184	-3.556	**
PGGD	118.49	.771	.651	.672	3.079	*
IN	2.17	.238	10.968	.148	1.375	

* significant at 10%
 ** significant at 5%
 *** significant at 1%

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CHAPTER IX (cont'd.)

Distinguishing Short-Run and Long-Run Effects

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Chapter X

Financial Markets and Monetary Policy

Increasing attention has been paid, both in academic research and in government, to the interaction of financial markets with the real economy. As a reflection of this our model has a more elaborate financial sector than has been traditional for models of its size. The main task of this sector is to determine the corporate bond and commercial paper rates, which are used in the fixed investment and income distribution sectors.

Most serious work on monetary econometrics is quite recent. This sector relies heavily on the work of Frank de Leeuw (1969), Patric H. Hendershott (1968), and a group at M.I.T., the Federal Reserve, and the University of Pennsylvania, who have constructed an elaborate model of the U.S. economy. We have drawn on an early version of the MIT-FRB-Penn financial sector presented by de Leeuw and Gramlich (1968). Financial markets are complex and subject to frequent structural changes, thereby making it exceptionally difficult to model their basic structure. We are much less confident of the validity of most equations reported here than for most other chapters. We shall first discuss the data series employed in this sector and the relation between our model and monetary policy as implemented by the Federal Reserve.

THE DATA

The first six exogenous variables listed in Table X.I are quarterly averages of monthly figures from the Federal Reserve Bulletin.

TABLE X.1

Variables Appearing in the Financial Sector

(Endogenous - Determined in the Financial Sector)

RESR	Required reserves of Federal Reserve member banks, billions of dollars.
RESF	Free reserves of Federal Reserve member banks, billions of dollars.
RCB	Yield on Moody's AAA-rate corporate bonds, per cent.
RCP	Yield on 4-6 month prime commercial paper, per cent.
RCL	Rate on prime commercial loans, per cent.
MTD	Time deposits at all commercial banks, billions of dollars.
RTD	Interest rate on time deposits at commercial banks, per cent.
MDDP	Private demand deposit liabilities of commercial banks, less interbank deposits, cash items in the process of collection, and Federal Reserve Float (adjusted demand deposits), billions of dollars.
MCL	Commercial and industrial loans at all commercial banks, billions of dollars.

(Endogenous - Determined Elsewhere in the Model)

INBIN	Change in business inventories, billions of dollars.
BDIV	Corporate dividend payments, billions of dollars.
YGNPI	Gross national product, billions of dollars.

(Exogenous Variables)

RRQD	Weighted average of required reserve ratios against demand deposits of Federal Reserve member banks, fraction.
RRQT	Reserve requirement ratio against time deposits of Federal Reserve member banks, fraction.

RTDMX	Legal maximum rate payable on commercial bank time deposits held six months or more, per cent.
DR	Federal Reserve Bank of New York discount rate, per cent.
MDDFG	Government demand deposits at all commercial banks, billions of dollars.
RESU	Unborrowed reserves of Federal Reserve member banks, billions of dollars.
DQ2	Dummy for second quarter, equal to 1.0 only in second quarters.
DQ3	Dummy for third quarter, equal to 1.0 only in third quarters.
DQ4	Dummy for fourth quarter, equal to 1.0 only in fourth quarters.

All National Income Figures are seasonally adjusted at Annual Rates

The Federal Reserve has four basic types of policy tools at its disposal. First, it can fix some of the interest rates on which banks and individuals base their decisions. For instance, the rate of interest on checking accounts is fixed (by law) at zero. The two principal non-zero interest rates determined by the Federal Reserve are present in this sector; they are DR and RTDMX. It should be noted, though, that both these rates are adjusted by the Federal Reserve passively as well as actively. Both rates are often moved to keep them "in line" with other interest rates, rather than to cause changes in financial market conditions. Since it is not at all clear how to separate movements of these rates into endogenous and exogenous components, we follow other authors and treat these two rates as exogenous.

The second set of tools the Federal Reserve can use is the reserve requirements ratios of member banks. Different classes of banks may have different required reserve ratios, but it would unduly complicate our model to take this into account. Since the fractions of deposits in each class of bank are nearly constant, we lose almost no realism by summarizing the reserve requirements policy parameters as RRQD and RRQT.

The third tool of the Federal Reserve is more frequently utilized than the first two. The Federal Reserve buys and sells government securities on the open market. A purchase of securities adds to the unborrowed reserves of member banks, while a sale reduces reserves. Such purchases and sales, called "open market operations", are undertaken on a daily basis. We represent this tool in our model by the policy variable RESU.

It should be noted that while RESU is treated as exogenous, what the Federal Reserve System controls to the last dollar is its own portfolio of

government securities, not unborrowed reserves. The difference between unborrowed reserves and Federal Reserve holdings of government securities consists of the gold stock, currency, and minor items such as Treasury and cash accounts and float, which sometimes fluctuate sharply from week to week. Treating RESU as a policy variable amounts to assuming that the Fed can, and often does, offset quarterly movements in the gold stock and currency.

Finally, the Federal Reserve can limit member bank borrowing. The Fed is not required to lend reserves to banks that seek to borrow them, and one often hears that the Fed "frowns" at borrowing or exercises "moral suasion". One popular text puts the matter as follows:

A Reserve bank rarely refuses to lend to a member bank that is facing an actual or prospective deficiency in its reserves. But after making a short-term loan to a bank, it studies the situation carefully. If it finds that the bank has borrowed too often, too continuously, too much, or for improper reasons, it may advise the bank to contract its loans or sell securities in order to reduce or retire its borrowings. It may even go as far as to refuse to renew the loan, and in extreme cases it may suspend the bank's borrowing privilege.

Federal Reserve officials could, of course, attempt to regulate the volume of member-bank borrowing by varying their own attitudes toward lending, being very strict at some times and more liberal at others. Though this is done to some extent, it is not a very flexible or effective instrument.*

Since it is virtually impossible to quantify Federal Reserve attitudes toward borrowing, we follow previous work and exclude this policy instrument from our model. Thus total bank reserves are determined by banks' demand for free reserves - defined below - and by the level of unborrowed reserves supplied by the Fed.

* Lester V. Chandler, (1964), pp.234-5.

Turning to endogenous variables, RESR is included among them because it depends on the level of demand and time deposits, even though reserve ratios are of course exogenous. All endogenous variables with the exception of RTD are quarterly averages of monthly figures taken from the Federal Reserve Bulletin. There are no published quarterly figures for time deposit yields, but an annual series can be computed from data in the annual reports of the Federal Deposit Insurance Corporation. De Leeuw (1965) and Hendershott (1968) have estimated quarterly series by fitting equations to the annual data and interpolating. Our RTD series is deLeeuw's until the first quarter of 1955, Hendershott's from there until the second quarter of 1968, and our own linear interpolation through the end of 1968. As rates were near their legal maximum in this last period and consequently did not vary much, more sophisticated interpolation procedures would have yielded little advantage.

The financial sector thus contains five monetary policy instruments under the control of the Federal Reserve: DR, RTDMX, RRQT, RRQD, and RESU. We could carry out simulations of this sector in isolation, not allowing for feedbacks from the rest of the economy, and examine the impact of these changes in these parameters on various endogenous variables. Such simulations are presented for an early version of the MIT-FRB-Penn financial sector in de Leeuw and Gramlich (1968), pages 13-16. It seems more sensible, however, to examine the impact of monetary policy in the economy as a whole, and we do this in Chapter XII.

THE EQUATIONS

Monetary economics typically divides the economy into three sectors: commercial banks, the non-bank public, and the government (which includes the Federal Reserve System). Based on market interest rates, requirements for transactions, current and expected income, wealth, and tastes, the non-bank public decides in what form to hold its wealth. An increase in interest rates, for instance, will lead to a desire to shift away from currency and demand deposits into assets which yield a positive return. This sector also decides on the level of bank loans it desires. Commercial banks decide at what terms they will accept time deposits and issue commercial loans as well as the amount of government securities they are willing to hold. The government, as mentioned above, makes reserves available to the banking system, fixes reserve requirements for Federal Reserve member banks, and sets certain interest rates.

The non-bank public can choose from among a variety of financial assets as stores of wealth. Besides demand and time deposits, there are common and preferred stocks, a variety of corporate and government bonds, commercial paper, and savings and loan shares. (Non-bank financial intermediaries such as savings and loan associations and mutual savings banks should be distinguished from the rest of the non-bank public in a less aggregative treatment.

A rigorously correct analysis must consider all assets and liabilities for one simple reason. For any economic agent or sector, assets always equal liabilities. Failure to consider this budget constraint explicitly can lead to rather fundamental mis-specification. Brainard and Tobin (1968) and Tobin (1969) have stressed this point most convincingly. Yet we do not incorporate the sectoral budget constraints into our model.

We offer two reasons for this omission. First, a model considering all financial assets and liabilities and all corresponding interest rates would be excessively complicated for the objectives of this monograph. The evidence suggests that unless complicated estimation methods are employed, such an approach will not yield satisfactory statistical results.* Second, our aim throughout this text is to describe the state of the art of econometric model building. And no comprehensive financial models exist (as far as we know) which adequately consider sectoral budget constraints.

This sector focuses on a limited number of structural relationships, with heavy emphasis on bank behavior. Our first equation is the identity relating the three categories of bank reserves listed in Table X.1:

$$(10.1) \quad \text{RESF} = \text{RESU} - \text{RESR}.$$

The other equivalent way to define free reserves is as the difference between banks' excess reserves and reserves borrowed from the Fed. The first stochastic equation relates RESR to the level of commercial bank deposits.

Seven behavioral equations comprise the remainder of the sector. We estimate supply and demand equations for commercial loans and time deposits and a demand equation for demand deposits. (There is no supply equation for this asset, since the interest rate on demand deposits is fixed by law and no useable time series exist on the level of checking account service charges.) A demand equation for free reserves and an equation relating the commercial paper and corporate bond rates round out the sector.

*Gramlich and Kalchbrenner (1969) present some estimates of the demand for liquid assets which take into account the budget constraint of the non-bank public. Even in this limited case, sophisticated assumptions and estimation techniques were required to yield sensible results.

The nine equations of this sector determine the nine endogenous variables listed in Table X.1. This sector differs from the others presented so far in that it is not the case that each endogenous variable is the dependent variable in one structural equation. Free reserves appear on the left of two equations, while RCP appears only as an independent variable. To employ these equations in simulation, we shall have to re-normalize some of them. That is, we shall have to re-write at least one equation so that RESF and RCP are the dependent variables of one and only one equation each. This process is discussed in the policy simulations in Chapter XII. We now turn to examinations of the eight stochastic equations in the sector.

a. Required Reserves

If MDDP, MDDFG, and MTD were deposits only at Federal Reserve Member banks, and if RESR did not include required reserves held against inter-bank deposits, we would have the following identity:

$$\text{RESR} = \text{RRQD} (\text{MDDP} + \text{MDDFG}) + \text{RRQT MTD}.$$

But none of the if's in the last sentence hold. Not all deposits are held with Federal Reserve member banks, though the fraction held there is reasonably stable. In addition, some required reserves are held against inter-bank deposits, which are netted out of MDDP. To approximate these influences on total reserves, the following equation was estimated based on the work of de Leeuw and Gramlich (1968), and de Leeuw (1969).

$$\begin{aligned}
 (10.2) \quad \text{RESR} &= 1.4057 + .8429 \text{ RRQD (MDDP + MDDFG)} \\
 &\quad (2.96) \quad (29.2) \\
 &+ .6089 \text{ RRQT (MTD)} - .3266 \text{ DQ2} \\
 &\quad (31.3) \quad (6.52) \\
 &- .2109 \text{ DQ3} + .1947 \text{ DQ4} \\
 &\quad (4.22) \quad (3.89)
 \end{aligned}
 \quad
 \begin{aligned}
 R^2 &= .995 \\
 SE &= .132 \\
 DW &= 2.03
 \end{aligned}$$

The second and third coefficients indicate that about 84% of demand deposits and 61% of time deposits are held with Federal Reserve member banks.

b. Commercial Loans

The demand function for commercial loans follows the work of Goldfeld (1969). Business loans ought logically to depend on rates of interest and on business' need for working capital. An equation all of whose coefficients were theoretically acceptable was hard to find. The best obtained was:

$$\begin{aligned}
 (10.3) \quad [\text{MCL} - \text{MCL}(t-1)] &= -.0569 \text{ MCL}(t-1) - 1.0935 \text{ RCL} \\
 &\quad (3.71) \quad (6.01) \\
 &+ .4671 \text{ RCP} + .0655 \text{ INBIN} + .4961 \text{ BDIV} \\
 &\quad (3.16) \quad (3.11) \quad (5.95) \\
 &- .6596 \text{ DUCD} \\
 &\quad (2.28)
 \end{aligned}
 \quad
 \begin{aligned}
 R^2 &= .764 \\
 SE &= .488 \\
 DW &= 1.99
 \end{aligned}$$

All the coefficients have the expected sign, and all are significant. But the implied mean lag of 16.5 quarters is excessive, and the coefficient of BDIV is also larger than theory suggests is reasonable. Specifications that involved scaling the dollar variables by YGPI, as described below, failed rather miserably, as did log-linear equations. Equation (10.3) does fit well, but it clearly is quite imperfect. It illustrates the conundrums of dealing with complicated phenomena in a simple manner but it is ingenuity in doing that well which constitutes the substance of macro-econometrics.

The commercial loan rate is fairly sticky, but it eventually responds to movements in other interest rates. The supply equation for commercial loans used in this model is the following:*

$$\begin{aligned}
 (10.4) \quad \log(RCL) = & \begin{array}{ccc} .3106 & + & .3060 \log(RCB) + .0850 \log(RCP) \\ (7.97) & (5.58) & (5.62) \end{array} \\
 & + .4618 \log(RCL(t-1)) \\
 & (7.65)
 \end{aligned}$$

R^2	=	.982
SE	=	.0202
DW	=	1.31

The Durbin-Watson suggests the presence of serial correlation, but the other statistics are acceptable. The sluggishness of RCL does not appear in the mean lag, which is less than a quarter, but rather in the fact that the slow-moving long rate, RCB, is much more important here than the more volatile short rate, RCP.

c. Time Deposits

The public's demand for time deposits should be affected by the yield on time deposits as against the yield on other assets, and by the

* Linear versions of this equation are in Patric H. Hendershott, (1968), page 56, Eqns.(22) and (23). Also Frank de Leeuw and Edward Gramlich, (1968), page 31, Eqn.(10).

level of income. As usual, gradual adjustment is present in the form of first order lags. We have followed de Leeuw (1965) here in scaling the dollar variables by dividing by income. This forces the income elasticity to unity and removes the effects of the changing scale of the economy. The equation form finally adopted is standard.* The equation that performed best was:

$$\begin{aligned}
 (10.5) \quad (MTD/YGNPI) = & .009591 + .9490 [(MTD)(t-1)/YGNPI(t-1)] \\
 & (5.61) \quad (49.7) \\
 & + .005878 RTD - .001650 (RCP+RCB) \\
 & (6.99) \quad (8.00) \\
 & - .1957 [(YGNPI-YGNPI(t-1))/YGNPI(t-1)] \\
 & (11.3)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .0012 \\
 DW &= 1.75
 \end{aligned}$$

All terms have the expected sign and are significant; the fit is excellent. The implied mean lag of about 19 periods shows that time deposits are adjusted slowly. The last term in (10.5) indicates that when income increases, time deposits are temporarily drawn down, to pay for increased durables consumption and business outlays for inventories and to meet other short term financial needs.

The supply function for time deposits determines RTD, the average rate at which banks will accept such deposits. As banks are unwilling to make frequent sizeable changes in the time deposit rate, RTD should lag its determinants. The equilibrium time deposit rate should be positively related to other interest rates, and it should be higher the greater the ratio of

* Hendershott (1968), pg.53, Eqn.(19); and de Leeuw and Gramlich, (1968), pg.31, Eqn.(4); and de Leeuw (1969), pg.277, Eqn(9.3). This equation is like de Leeuw and Gramlich (1968), pg.31, Eqn.(3).

commercial loans to deposits, since this ratio reflects the pressure on banks to obtain loan funds. The existence of an effective ceiling rate, RTDMX, limits the long-run response of the observed rate to less than the desired response. In contrast, banks should adjust the time deposit rate quite rapidly to changes in the effective ceiling. A decrease in the ceiling rate forces some banks to lower their rates immediately, and an increase should lead banks that would have had higher rates in the absence of the ceiling to raise their rates quickly.

Hendershott's (1968) quarterly RTD series was obtained by fitting a supply relation to annual data and interpolating. Our equation for RTD is basically the one he used:*

$$\begin{aligned}
 (10.6) \quad RTD = & - .1581 + .9490 RTD(t-1) + .2977 RTDMX \\
 & (2.64) \quad (41.8) \quad (8.63) \\
 & - .2878 RTDMX (t-1) + .0140 RCB \\
 & (8.28) \quad (.631) \\
 & + .6162 [(MCL/(MTD+MDDP)) + (MCL(t-1)/MTD(t-1)+MDDP(t-1))] \\
 & (2.34)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .0367 \\
 DW &= 1.85
 \end{aligned}$$

These estimates are all in accord with our expectations except the coefficient of the corporate bond rate. But since this term was also insignificant in Hendershott's annual equation, and since no other interest rate performed any better, it was decided to retain RCB in the equation.

* See Patric H. Hendershott (1968), pp.49-52, 62-65. The closest equation is pg.52, equation (17).

d. Demand Deposits

The demand deposit equation, in logarithmic form, incorporates the classic elements of the Keynesian money demand equation in the format of a simple distributed lag, since money holdings cannot adjust instantaneously to changes in its determinants. The opportunity cost of holding cash is "the" rate of interest, here represented by the geometric mean (because the equation is log linear, following the MIT-FRB-Penn (1968) formulation) of the time deposit rate, RTD, and the commercial paper rate, RCP, both of which are low risk interest bearing close cash substitutes for individuals and businesses.* GNP represents the classical demand for means of payment.

$$(10.7) \quad \log MDDP = .0604 - .0521(\log RTD + \log RCP)/2 + .1429 \log YGNPI \\ (1.15) \quad (9.79) \quad (17.79) \\ + .8119 \log MDDP(t-1) \\ (66.12)$$

R^2	=	.9969
SE	=	.0048
Antilog SE	=	1.0048
DW	=	1.77

This is one of the few equations estimated with an autoregressive correction to allow for serially correlated errors. **

* See Tobin (1956)

** The untransformed equation is:

$$\log MDDP = - .2604 - .0407(\log RTD + \log RCP)/2 + .0758 \log YGNPI \\ (1.86) \quad (6.59) \quad (3.33) \\ + .9653 \log MDPP(t-1) \\ (17.49) \\ R^2 = .9960; \quad SE = .0055; \quad DW = 1.10$$

The transformation parameter estimate of first order autoregression was .6, to the nearest tenth, using the standard Hildreth-Lu (1960) estimating procedure. (see Christ (1966), pp.481-488.)

Partly we made this correction in line with MIT-FRB-Penn procedures because serial correlation is clearly present. But the main reasons were theoretical. The implied long-run income elasticity of demand of the ordinary least squares estimate exceeded two, which is far larger than any other reasonable estimation results have shown. The transformed income elasticity of .75 is within the range of acceptable estimates which cluster in the neighborhood of unity. Furthermore, the lag weight of .96 in the untransformed version suggests an excessively long adjustment period, while the transformed equation at .81 is much more reasonable. Considering the crude state of most monetary econometrics, this equation has acceptable theoretical and statistical properties.

a. Free Reserves

Free reserves are an extremely volatile series, and we do not explain them well, nor do others. Desired free reserves should depend positively on the discount rate, which represents the cost of borrowing reserves, and negatively on market interest rates, which represent the opportunity cost of holding excess reserves. Actual free reserves should adjust gradually to the equilibrium level. To eliminate the effects of changes in the scale of the banking system, reserves were scaled by dividing by lagged deposit liabilities. Finally, changes in unborrowed reserves and commercial loans, both essentially exogenous to the banking system, will cause temporary changes in the levels of free reserves. Following de Leeuw (1969) p.277, Eq. (9.4) and de Leeuw and Gramlich (1968), page 21, Equation (2), which followed the earlier work of Meigs (1962), our free reserve equation is:

$$(10.8) \quad [\text{RESF} - \text{RESF}(t-1)] / [\text{MDDP}(t-1) + \text{MTD}(t-1)] = .0014 \\ (3.72)$$

$$- .3653 [\text{RESF}(t-1) / (\text{MDDP}(t-1) + \text{MTD}(t-1))] + .0017 (\text{DR} - \text{RCP}) \\ (4.06) \quad (2.87)$$

$$+ .5425 [\text{RESU} - \text{RESU}(t-1)] / [\text{MDDP}(t-1) + \text{MTD}(t-1)] \\ (4.60)$$

$$- .1379 [\text{MCL} - \text{MCL}(t-1)] / [\text{MDDP}(t-1) + \text{MTD}(t-1)] \\ (3.02)$$

$$\begin{aligned} R^2 &= .544 \\ \text{SE} &= .0012 \\ \text{DW} &= 2.21 \end{aligned}$$

Given deposits, the last two terms imply that a one dollar increase in unborrowed reserves will increase free reserves 54¢ in the current quarter. In equilibrium, of course, the ratio of free reserves to deposits will be unaffected; both deposits and free reserves will expand. Similarly, an increase of one dollar in commercial loan demand causes a transient fall of 14¢ in free reserves. The mean lag of Equation (10.8) seems to be rather short, only 1.7 quarters, implying more rapid adjustment on the part of the banking system than had been anticipated.

f. Corporate Bond Rate

The final behavioral equation in this sector relates the corporate bond rate to the commercial paper rate. It is well known that in perfect capital markets the long-term rate of interest will always be the average of future expected short-term rates.* This is because borrowers can either borrow long or borrow short and repeatedly re-finance, and lenders can either buy long-term bonds or a sequence of short-term bonds. In reality,

* See Friedrich A. Lutz (1940), reprinted in Readings in the Theory of Income Distribution, Selected by a Committee of American Economic Association, Richard D. Irwin, Inc., Homewood, Illinois 1951; and Hicks (1939), Ch.XI.

borrowers would rather not have to worry about refinancing, since future short rates are uncertain, they thus are willing to pay a premium in order to sell long-term bonds. There is, therefore, a tendency for the long rate to be consistently above the short rate, a phenomenon called "normal backwardation" by J.M. Keynes.*

We assume that expected future short-term rates are a distributed lag function of past observed short rates.** Of the several equations attempted, the best equation on statistical grounds was the following:

$$(10.9) \quad \log(RCB) = .1281 + .0561 \log(RCP) + .8656 \log(RCB)(t-1)$$

(2.99) (2.76) (20.5)

$$\begin{aligned} R^2 &= .965 \\ SE &= .0286 \\ DW &= 1.50 \end{aligned}$$

The mean lag of equation (10.9) is a little over 6.4 quarters, rather long. The equilibrium relation estimated between RCB and RCP is approximately

$$(10.10) \quad RCB = 2.6 RCP^{.42}.$$

The significance of the constant term in (10.9) serves to refute the hypothesis that the multiplicative constant in (10.10) is equal to unity. Similarly, we were able to refute the null hypothesis that the exponent is unity.

* See Keynes (1930), Vol.II, p.135, and Hicks (1939), pp.146-147.

** See Franco Modigliani and Richard Sutch (1966) and references cited therein.

The financial sector of this model consists of equations (10.1)-(10.9). This set of equations is clearly highly non-linear and also highly simultaneous, since several endogenous monetary variables appear in each equation. As mentioned previously, simulations appear in policy applications in Chapter XII.

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Chapter X - Financial Markets and Monetary Policy

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Chapter XI

Government Demand, Fiscal Policy, and Budgetary Accounts

Quarterly data only permits a condensed portrayal of government expenditures by level of government or by type of expenditure. Hence our model's government sector, in which government demand for goods and services is determined, is quite rudimentary.

We shall begin by presenting the National Income and Product Accounts' including government receipts as well as expenditures and relating it to our model. This discussion should give the reader some notion of how fiscal policy operates within the present model. The next section presents and discusses the data series employed in this sector. The final section discusses the three equations making up the government sector.

The Government Budgetary Accounts

Table XI.1 exhibits the structure of the government accounts, along with the values of the various components in 1969. This table also gives the symbols used in this text for the series involved.

Equations for the four major receipt categories were presented in Chapter VII. The various levels of government set a number of tax rates which, along with activity in the private sector, determine actual receipts.

TABLE XI.1

General Government Receipts and Expenditures: 1969Billions of Current Dollars*Receipts

Personal Tax and Non-tax Payments (GRPTX)		
Federal	95.9	
State & Local	21.4	117.3
Corporate Profits Tax Liabilities (BCPT)		
Federal	39.2	
State & Local	3.5	42.7
Indirect Business Tax & Non-tax Liabilities (BIBT)		
Federal	19.1	
State & Local	66.1	85.2
Contributions for Social Insurance (GRFICA)		
Federal	46.4	
State & Local	7.1	53.5
<u>Total Net Receipts</u>		<u>298.7</u>

Expenditures

Compensation of Employees (YGGPI)		
Federal: Defense	32.0	
Federal: Other	10.0	
State & Local	61.6	103.6
Purchases of Goods & Other Services (GGDI)		
Federal: Defense	46.6	
Federal: Other	11.6	
State & Local	49.2	107.4
Transfer Payments to Persons (GETRFP)		
Federal	50.0	
State & Local	11.5	61.5
Net Federal Transfer Payments to Foreigners (GETFF)		2.0
Net Interest Paid (GEINT)		
Federal	13.1	
State & Local	11.5	24.6
Subsidies Less Current Surplus of Govt. Enterprises (GRSUB)		
Federal	4.6	
State & Local	- 3.6	1.0
<u>Total Net Expenditures</u>		<u>300.1</u>
<u>Surplus on National Income Accounts</u>		<u>- 1.4</u>

Source: Survey of Current Business, July 1970. Table 3.1, p.29, Table 3.3, p.30, and Table 3.11, p.35. Totals may not add exactly because of rounding error.

The equations for GRPTX and BCPT contain two such rates, CGBTRT and BCPTRT. No tax rate is present for BIBT, but it is easy to see how a general change in rates would change the corresponding equation.

Contributions for Social Insurance are affected by a number of policy parameters. Three of these (TXRSS, TMXEPE, and TXRUB) are employed in the equation for this variable. The model thus contains explicitly five tax rates which can be altered in fiscal policy experiments.*

Let us now turn to government expenditures. The first item in Table XI.1, compensation of government employees, is treated in the national income accounts as government's entire contribution to national product. It is called Gross Government Product. Since the output of government activities is seldom valued by market prices, it must be valued at the cost of inputs. The second category of expenditures consists of purchases from private sector businesses. The sum of these two categories is simply called Government Purchases of Goods and Services, and it appears on the product side of the National Income Accounts. Since government purchases can obviously be varied for reasons of fiscal policy, we shall examine their impact in the next chapter.

Net Federal Transfer Payments to Foreigners do not appear in the National Income Accounts, though they do affect Balance of Payments measures and, of course, government expenditures on national income account. This variable is treated as exogenous in this model.

* Since BIBT includes excise taxes, any experiments involving a mechanical alteration of the ratio of BIBT to Gross National Product would have to take account of tax shifting and the resultant price changes. As this is a very complicated business, no such simulations will be undertaken. See G. Fromm and P. Taubman, (1968), Chapter 3, with an Econometric

Transfer payments to Persons appears on the income side of the national accounts. Here as in the case of taxes, the governments involved determine rates, and actual payments are a function both of rates and of private activity. The only rate appearing explicitly in the equation for GETRFP is TVMOA, though changes in the benefit structure of unemployment insurance are easily incorporated in simulation.

Net interest paid by government, which appears on the income side of the national accounts as a special class of transfer payment, is not subject to government control in this model. It depends on interest rates, determined in the financial sector, and on the level of interest-bearing government debt, taken as exogenous to the model. In reality, the level of total government debt is determined by government receipts and expenditures, and the division between interest-bearing and non-interest-bearing debt is determined by monetary policy. There is thus a government budget constraint, which may have important implications for stabilization policy.* But the relevant receipts and expenditures are not those measured in the national income accounts; to build this constraint into our model we would need to consider alternative measures of government receipts and expenditures.**

* See C. F. Christ (1967).

** The series necessary to do this are not available on a quarterly basis. Even if data were no problem, consideration of the government budget constraint would be highly complicated. On the relations between the various government budget concepts, see J. Scherer (1965). (Reprinted in W. L. Smith and R. L. Teigen, Readings in Money, National Income, and Stabilization Policy (Homewood, Ill.: R. D. Irwin, 1965).

The final category of expenditures, net subsidies of government enterprises, is also considered a transfer payment, and it appears on the income side of the national accounts. This quantity is taken as exogenous.

This model thus contains a number of tax and expenditure rates which can be varied for purposes of fiscal policy. Also, government compensation of employees and purchases from the private sector can be changed for stabilization reasons. A wide variety of fiscal policy experiments can be performed with this model, some of which will be investigated in Chapter XII.

THE DATA FOR GOVERNMENT DEMAND

The variables used in the government demand sector are shown in Table XI.2. Government purchases from the private sector is defined as total government purchases of goods and services minus government compensation of employees. The latter is equal, of course, to gross government product. Given total government purchases of goods and services in current and constant dollars from the Survey of Current Business along with YGGP and YGGPI, from the same source, GGD and GGDI are obtained by subtraction. The implicit deflator PGGD is then computed as $(100 \text{ GGDI}/\text{GGD})$.

WRPVT is discussed in Chapter IX. Briefly, it is calculated by dividing private compensation of employees (YPRCE) by an unpublished Bureau of Labor Statistics series for total manhours worked in the private economy (PEHRS). LEGM and LEGC are quarterly averages of monthly figures from Employment and Earnings. All other series shown in Table VII.1 were either taken directly from the Survey of Current Business or computed from series taken from the Survey, as outlined above. All flow variables are seasonally adjusted quarterly totals expressed at annual rates.

TABLE XI.2Variables Appearing in Government Demand Sector

(Endogenous Variables - Determined in Government Demand Sector)

GGD	Government purchases from the private sector, billions of constant dollars.
YGGPI	Gross government product, billions of current dollars.
YGGP	Gross government product, billions of constant 1958 dollars.
GSRP	Government Surplus on National Accounts basis, billions of current dollars. (Endogenous Variables - Determined Elsewhere in the Model)
WRPTV	Gross private wage rate, dollars per manhour.
PGGD	Implicit price deflator for government goods demand, 1957-59 = 100.
PGGP	Implicit price deflator for gross government product, 1957-59 = 100.

(Exogenous Variables)

LEGM	Government military employment, millions of persons.
LEGC	Government civilian employment, millions of persons.
GGDI	Government purchases from the private sector, billions of current dollars.
GETFF	Net Federal transfer payments to Foreigners, billions of current dollars.

All National Income Figures are seasonally adjusted at Annual Rates

THE EQUATIONS for Government Demand

We take current dollar government purchases from the private sector as exogenous. If a finer breakdown of the government expenditure accounts were available on a quarterly basis, some components of this total could probably be explained within the model.*

Given GGDI, GGD is determined by

$$(11.1) \quad GGD = GGDI (100./PGGD),$$

with PGGD determined in the wage-price sector. Similarly, given YGGPI, YGGP is determined by

$$(11.2) \quad YGGP = YGGPI (100./PGGP),$$

and PGGP is also determined in the wage-price sector.

Instead of treating YGGPI, government compensation of employees in current dollars as exogenous, we take government civilian and military employment as the policy variables, since the wage paid by government is determined by competitive remuneration rates in the private sector. If we suppose that the government wage rate adjusts gradually to the rate paid in the private sector, and if an allowance is made for differences in the average level of government civilian and military wages, we are led to the following equation:

$$(11.3) \quad YGGPI = \underset{(6.63)}{.2509} (WRPVT LEGC) + \underset{(4.36)}{.4554} (WRPVT LEGM) \\ + \underset{(49.0)}{.9803} YGGPI (t-1) \quad \begin{array}{l} R^2 = .9996 \\ SE = .335 \\ DW = 1.62 \end{array}$$

* See L. C. Thurow (1969), for an example of what can be done with annual data. Also, Roger E. Bolton (1969), E. M. Gramlich (1969), and C.D. Phelps (1969).

Terms like (TIME WRPVT LEGC) failed consistently; no support was found for any trend in the ratio of government to private wage rates.

Equation (11.3) indicates higher military than civilian wages; the difference in the coefficients is significant at the 1% level. This equation provides a remarkable fit to the data. The standard error of \$335 million is tiny next to the sample mean of YGGPI of \$52.3 billion. The mean lag of 9.9 quarters seems suspiciously long, but perhaps only people who have not spent much time working for the government can say that.

Finally, we have the identity determining net government surplus;

$$(11.4) \quad \text{GSRP} = (\text{GRPTX} + \text{BCPT} + \text{RIBT} + \text{GRFICA}) \\ - (\text{YGGPI} + \text{GGDI} + \text{GETRFP} + \text{GETFF} + \text{GEINT} + \text{GRSUB})$$

This equation follows directly from Table IX.2.

The government sector thus consists of identities (11.1), (11.2), and (11.4), along with the single behavioral equation (11.3). As the latter fit the data so well, it did not seem that much information would be gained by simulating it over the sample period or over the year 1968. Consequently, no simulation experiments were performed with this sector.

CHAPTER XI - Government Demand, Fiscal Policy and Budgetary Accounts

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